School Science

VOL. III.] MARCH, 1904. [NO. 9

IS THE COURSE FOR COLLEGE ENTRANCE REQUIREMENTS BEST FOR THOSE WHO GO NO FURTHER?*

BY JAMES G. NEEDHAM,

Professor of Biology, Lake Forest College.

The question is an old one. Is there conflict or harmony of interests between secondary and higher education? Should the high school student be laying foundations for future study, or should he be doing work that is complete in itself, so far as it goes; or, may he not arrive at a maximum of present utility while laying satisfactory foundations for future studies? I should prefer to discuss the question the other end about: for the need of the majority is the constant term involved—fairly constant, at least, since that need will change only with the slow alteration of environment—while the entrance requirement is a much more variable quantity. Let us ask then: Is not the course in biology that is best for the student who ends his studies with the high school a good and satisfactory preparation for college?

When the struggle for existence between subjects now contending for place in the school program shall have worked itself out we will probably know better what is best for the majority "who go no further." Now we must needs exercise foresight, while hindsight will be much clearer. We may gain some hints of things to come by comparing the situation of these newer subjects with the state of those that have reached the end of

*Address delivered before the Biological Section of the Central Association of Science and Mathematics Teachers in Chicago, Friday, November 27, 1903. General subject of the meeting: Essentials of a High School Course in Biology.

the struggle and established themselves. The subjects now universally conceded a place in the school program, such as reading, writing, arithmetic, spelling, grammar, geography, etc., stand in marked contrast with some of the newer subjects as respects articulation. These older subjects are orderly, consecutive and complete in themselves; the student drops any of them anywhere without loss—with only gain for what he has had—even though, for example, he stop between short and long division. The list of such studies is longer than it once was, and it may well be that other subjects will come to take their places as essentials, as they demonstrate the same degree of educational efficiency and adjust themselves in orderly and progressive sequence.

It must be admitted at once that at present there is no biological program. Studies of living things begin in some places in the kindergarten; in some, in the grades; in some, in the high school; in some, in the college; and in some, they do not begin at all. In some, they are continuous; in some, interrupted; in most, there is little effort at articulation. The unsettled state of our subject is remarkably evidenced in three different ways: (1) In the rapid shifts of emphasis as to what shall be taught; (2) in the diversity of high school textbooks, and (3) in the indefiniteness of the college entrance requirements.

1. The shifts of emphasis are due chiefly to the fact that most of our nature study has been handed down from above instead of growing up from below. High school and normal school zoölogy and botany have too often been handed down ready made by university professors. In my own college days it was all systems of classification they were handing down. In my college days it was all anatomy; now it is nearly all ecology. It is now hardly more than a decade since many teachers, newly returned from college or normal school, where their zoölogical training had consisted in dissecting a cat, were trying the same course they had taken, without dilution or alteration, on the little, innocent children. This did not last long, however, for the body politic is more or less resistent to the germs of educational diseases; but it lasted long enough to leave in the mind of the public an unsavory impression of zoölogy, not yet entirely lived down.

2. The diversity of textbooks is very great, both in subject matter and method. Some of the recent ones are all reading—storiettes about animals and plants; some are all dissecting; some are all keys and descriptions for determining of forms; some are all physiology and experimentation; some are all ecology, and some are admixtures of some or of all these things, and if any one wishes to learn whether these different things are considered pedagogical equivalents, just let him read the prefaces of these books. This diversity is the result of trying to fit one of the most extensive subjects with which the human mind has to deal into one of the smallest niches in the high school program. Each author appears to have included what he has been able to get in satisfactorily and has lopped off the remainder.

3. The usual college entrance requirement in biology at present is one year of *some laboratory science!* Surely this is broad enough to meet the demands of pioneer conditions.

What we have settled among ourselves appears to be that it is worth while to study living things at first hand. Since we may not do more, let us congratulate ourselves that we have progressed thus far and pull ourselves together for a new start.

What of biology shall be taught in the high school? Is not this a pedagogic question? Yes, as are all questions of fitting of subject matter to the receptivity of the developing mind. Is it not also a scientific question? Yes, as science must adjudge the worth of the subject matter. But biological education is more than either pedagogy or science—more than details of instruction, or biological facts and phenomena. It must be in the long run orderly and progressive development toward fitness for the activities of life. The place and portion of biology in the curriculum will not be determined by the dictum of the colleges, or the preferences of the schools, or the methodology of philosophers, but by the operation of natural laws, chiefly the law of natural selection. If biological teaching survive in the high school or anywhere else, it will survive by reason of its fitness as a part in the preparation for life. Therefore, we must never lose sight of the peculiarly intimate relation biology bears to human life. On the practical side, what other subject can compare with one whose chief practical applications are:

First. Living in this World—hygiene, in its very broadest application, including all personal control over the welfare of body and mind.

Second. Getting the materials of livelihood—agriculture, in its very broadest application, including all that relates to our dependence on the organic life of the world.

Third. Medicine—the healing art, sometimes mistakenly called the principal application of biology.

I will not mention the multitude of newer applications arising on every hand and making ever increasing demands for knowledge of the facts and principles of life.

Out of these relations there grow, I think, four incontestable reasons why every one should study biology:

1. To know animals and plants better. We have to deal with them in life. We should know how to protect our friends and combat our enemies among them, and to appreciate the place in the world of all of them. The ancient poetic vision of creation ends with the statement concerning every living thing, "To you it shall be for meat."

2. To know our environment better, not alone its economic, but also its æsthetic side; to know the charm of life, its wonderful beauty of color and form, its grace of motion, its adaptation to place and function. Here poets and artists and naturalists alike have found themes since the beginning of civilization.

3. To know ourselves better—possessors of animal bodies that are subject to the same laws, that are moved by the same instincts and that feel the same necessities as other animal bodies, and on the normal healthful activity of which all our possibilities of happiness and usefulness in life depend.

4. To know something of the development of life in the world, and thus to get acquainted with those general developmental principles which underlie modern methods of study in all departments of knowledge; which were first fully developed and are still best exemplified in the field of biology.

Now, it seems to me that keeping these matters in mind will help us to determine what are some of the things that should constitute part of the intellectual stock-in-trade of the average com-

ing citizen, who will go no further in formal studies than the high school. I will venture to name seven phases of biology, now more or less commonly studied, the value of which as parts of a high school course I consider already demonstrated.

1. *Elementary classification:* The systematizing of the random observations of nature study in the grades and of contact in life with living things. It need not be very extensive, and might about as well use common names as technical; but it should be a genuine gathering together of known forms into natural groups and a fixing of such groups by names. It will not matter much if, through lack of insight, some form occasionally gets into the wrong group, for such slips still occur with accomplished specialists. Classification naturally and properly follows hard upon the heels of observation, and only goes astray when it runs on ahead. Classification furnishes the handles by which we move all our intellectual luggage. Let us have just enough for our needs.

A modicum of collection making may be allowed here; if fondness is shown for it, it may even be encouraged in individuals and outside the allotted program; and the use of analytical keys should certainly be taught by a little practice. How many naturalists have begun their careers by making collections, and how great is the influence in the present day of the ever increasing number of manuals and handbooks that are spreading abroad the knowledge of living things.

For many years I have heard professional botanists railing against the old-fashioned course in flower analysis, but I want to testify that I once had such a course, and I have never had a better course in botany or in any other subject whatsoever. It was all nature study of the very best sort and full of the delights of discovery, and the worst that could be said of it is that it was one-sided and incomplete—not a very bad charge, considering the limitations of our knowledge and the immensity of the field.

2. *The study of living nature,* whether we call it old-fashioned natural history or new-fashioned ecology does not matter. In either case we mean the study of plants and animals in relation to their environment. This is the study of the phenomena of fitness. It is simple enough to interest the youngest mind, and profound

enough to have furnished the basis for our most important biological generalizations.

It should never be merely reading and talking about remote and wonderfully adapted creatures, but, instead, detailed and practical studies of the adaptation of common plants and animals. For instance, protective coloration should not begin with the Kallima butterfly, but with the grasshoppers and moths of the dooryard, and results should be secured that are as definite as those of the study of the anatomy of the grasshopper. Merely noting resemblance is not studying it. The pupil should record comparatively the details of the resemblance, whether general or specific, whether in form or color, how brought about, to what particular environment best fitted, the relative perfection of it, the differences in different animals, etc.

With all the emphasis that is placed on ecology in many recent high school books, it is astonishing how little attention is given to pointing a way for the inductive study of ecology on the part of students. It seems hardly to be recognized yet that ecological types are as common and as widely distributed as are morphological types, and that their study may be made to yield equally definite results. It is perhaps excusable, therefore, when teachers read the interesting discussions presented in these books, and instead of applying inductive methods to the study of the same subjects revert to anatomy for pedagogic results, or else lapse into textbook and recitation methods; but it is still painful and lamentable, and altogether unnecessary.

There are values of one sort growing out of the intensive laboratory study of a few types; these values have long been recognized. There are other and equal values growing out of the observation of nature in a great variety of forms and relations. These latter values a good ecological program will enable us to realize.

3. *A few practical, individual exercises in methods of economic procedure,* based on and necessitating a somewhat intimate knowledge of the structure, functions and habits of important animals and plants and their enemies—not the mere entertaining observations of nature study in the grades, such as feeding a frog on cutworms; such things should have been done already; but simple,

practical, economic experiments under natural conditions with the fundamental biologic facts and the desired practical results kept clearly in mind. I would include this, not as a sop to "practical folk," though it would in many cases make for solidarity between school and home, but because it is justified on good pedagogic grounds. The youthful mind is practical. Interest is sharpened and the details of scientific knowledge are better appreciated when things taught are recognized as constituting useful knowledge.

4. *The study of reproduction and development.* This is in a sense half of biology, for the place of a species on the earth is maintained if it (1) get a living, and (2) reproduce its kind. I deem the few local and sporadic attempts that have been made to exclude all consideration of reproduction from the high school course as an unworthy concession to near-sighted pseudo-pedagogy. For my own part I have always deemed it a privilege to bring to young people some real information as a basis for sane consideration of this much abused subject. Aside from the paramount importance of the subject biologically, I should regret to see this (oftentimes the only) gateway of practical knowledge shut before them. Furthermore, I am inclined to think that the teaching of these matters is needed as an antidote to the smut of the ancient classics and of English history. I judge the results of the teaching of this subject not by the attitude of the student when it is first broached, but by his attitude when the study is done.

Life history studies, it seems to me, are worthy of the greater part of the time spent on these matters, and to these may be added a modicum of embryology of the most elementary sort, preferably for us in the interior, on the eggs of some amphibian, and a brief, clear and straightforward presentation of the essential features of reproduction, illustrated in the lower forms of animal life and in plants.

5. *Physiology,* especially the physiology of organs. This already holds a secure and well merited place; so I but mention it in passing.

6. *The study of structure.* Anatomy, for a considerable period, held the field almost to the exclusion of every other phase of biological study. But with recognition of the fact that the

educational values of biology are far from being confined to the dissecting table, some of the anatomical work has had to go. We must forever give over the attempt to illustrate the whole gamut of evolutionary changes in a series of types. But we may retain enough of anatomy to be comparative, enough to illustrate kinship clearly, enough to illustrate differentiation, homology, analogy, etc. Let us give preference to external anatomy and the study of whole microörganisms, over internal anatomy and microtome sections. Other things being equal, let us give preference to the sort of work that the interested student may continue with his own resources after he has left the laboratory behind. And may we do all this with a maximum of fact and a minimum of terminology!

7. *Lastly,* there should be included the more *general conceptions* that have grown out of the consideration of biological facts and phenomena and that have taken their places in the world of thought. I mean evolution, with practical studies in the survival of the fittest; the biogenetic law, with practical detailed study of some illustration of the correspondence between ontogeny and phylogeny, etc. These should be introduced because they can not in justice be withheld rather than because the majority go no further. I would have them introduced, also, because some, who are accustomed to get their basis for thinking by more roundabout methods, are still maintaining that biology is a purely observational subject. These all but universal principles the world owes chiefly to biology and may rightly expect that teachers of biology will faithfully teach them and not withhold the indication of their wide applicability.

Let it be understood that these seven phases of the subject are not offered as a program; far from it. They are not topics for study, but matters to be emphasized in connection with any or all of the special topics to which they relate. I submit that among them is nothing that will not commend itself both for present value and for value as a basis for further progress in biology. I do not believe that any one is well equipped for intelligent participation in modern life if ignorant of these things. Without knowledge of them he will not know how to manage his own garden, his own table, his own appetites, his own emotions, or his own think-

ing. It is perhaps true that there are those in circles of culture ready to apologize for the mispronunciation of a Latin phrase, or for the admission of not having read Ivanhoe, or even Treasure Island, who would think nothing of it if one should call a whale a fish, or try to kill squashbugs by spraying them with paris green, or ask what beetles turn into. Indeed, our leading newspapers still publish several times a year the circumstantial details of one who, while drinking at a spring, swallowed tadpoles, and later coughed up frogs. But these things will not always be. On the other side of the matter, I would say for my own part that so far as knowledge goes, it is some little real and first-hand knowledge of just these seven aspects of biology that I should like to have the high school graduate equipped with when he presents himself for further work in college. It will have become sufficiently evident that, in my opinion, if the course that is best for life is not best for college entrance, it is so much the worse for the entrance requirement.

Even the few general topics I have named I would not at present *require* to be taught anywhere. I would merely recommend them. For while the science is so new, the field of possible studies so vast and the preparation of teachers so diverse, there is great danger that too much definiteness in a set program may curb initiative and curtail spontaneity. I would let the teachers of the present generation of pioneers do what they can do best to teach the rising generation to see and think, to know and love their environment, and to feel their kinship with the life of the world in body and spirit. Out of this work greater uniformity and better correlation will proceed naturally.

For pioneer conditions must pass. I once had a teacher of arithmetic who had a failing for the duodecimal system; that system had its beauties; and its educational utilities, also; but it has had to go. As it is no longer admissible to pasture one's cow on the common or to pick strawberries in any fence row, the time is sure to come when it will not be admissible for any teacher to teach what he pleases and when he pleases, according to the exigencies of his situation, the limitations of his knowledge or the prevailing fashion of his university. But it is this very freedom that allows the development of the possibilities of the subject; elimination will

come later. May it be natural elimination, and not the forced kind that education suffers when "men of violence take it by force."

What is best for life is not completeness, for that is unattainable; not so much great knowledge as a little knowledge rightly attained, with an appetite for more. One danger in programs is that knowledge will be the chief end sought. But another, and perhaps even greater danger, is that they will be arranged from the standpoint of the specialist without due regard to the standpoint of the learner. How often has it been forgotten already that we had fingers before forceps, eyes before lenses, lenses before microscopes, jackknives before scalpels, scalpels before microtomes. I have never found a truer statement of this matter than the following one from Prof. J. Arthur Thompson: "A circuitous course of study, followed with natural eagerness, will lead to better results than the most logical of programs, if that takes no root in the life of the student."

I can not help feeling that science teaching, while it has earned its place, has fallen far short of accomplishing that public good for which we may reasonably hope: the diffusion of honesty and directness of method and respect for the simple truth; the abandonment of dogmatism and superstition. Perhaps it is because of the essential conservatism of human nature, perhaps it is because this teaching starts too late and finds scant lodgment in soil already stocked with the notions of an unscientific age; perhaps it is because this teaching is not yet direct and forceful enough to take hold upon the life and to touch the springs of conduct. But ultimate failure in these respects would rest especially upon biology, because of the intimate relations it bears to the life of the people.

A PLEA FOR STUDENT LABORATORY WORK IN A FIRST COURSE IN PHYSICS.

SECOND PAPER.

BY W. F. MONCREIFF.

Professor of Mathematics and Astronomy, Winthrop College, Rock Hill, S. C.

In the first paper the general plan of work and its advantages were discussed. In this, objections to adopting the method will be considered.

To emphasize the reasonableness of the contention that individual laboratory work by the student is the only real way to present a first course in physics one might cite the fact that in most of our best schools it is the method practiced in most other natural sciences. Yet as far as can be ascertained not a high school or college in the South adopts this method in a first view of physics, and it is by no means as general in the North and West as the same method is in other similar subjects. The text-book and lecture-demonstration methods prevail in the South—the latter having taken precedence in the last few years. This precedence is an advance and a hopeful sign, but the present prevailing method falls short of individual laboratory work by all that is implied in the difference between seeing a thing done or hearing about how it should be done and *doing it yourself*—a difference which need only be stated to be appreciated, even by the layman.

But to the objections. They are assumed to be all practical in character. In the blaze of twentieth century psychology and pedagogy few would hazard a theoretical objection. Practical objections can usually be met in practical ways if those concerned have the determination to work out these ways and the good sense to adopt them. One of the most obvious and prevalent objections is that physical apparatus is very expensive and that our high schools have no equipment and our colleges poor ones. This is admitted; but it is no argument against the method of experimental work by the student. The roughest devices used by the student are incomparably better than the most elaborate ones used by the teacher, or descriptions of how the latter are used by some unheard-of scientist. For the purposes of a first course rough apparatus, devised

largely by teacher and student, is really superior to refined. Because (1) the raw student can not manipulate, or comprehend the action of, complicated machines; (2) the time necessary in their use is too great; (3) the rough approximations which he derives through the use of rough implements gives him a better insight into how physical science has grown up and a truer grasp of the relation between concrete work and abstract theory.

Another objection is lack of preparation on the part of teachers. In one sense this objection may have some force as related to teachers in many high schools and in some colleges. But in a higher and better sense it has little force in any case. Anyone who realizes, as all true teachers do, that, as a teacher, he exists solely for the best good of his students, and not for his own ease, profit, or glorification, and who has the pluck and determination necessary to real success can achieve it by use of this method. For such, special training is not necessary, though, of course, it would be helpful. But those who wish an easy time, and those who measure the amount of work they should do by the dollars and cents they get for it, and those who have a small reputation to guard, which they hope by careful nursing—never falling into error or getting into close places, never making a mistake or having to admit that they do not know—may grow a bit, for *such* the method here advocated would be hazardous. The only safe policy for these *people* (not teachers) is rigid adherence to a memorized text. And the only safe policy for the boards who have employed them, provided they regard the good of the student as the real end of teaching, is to get rid of them at once. Independent work of any kind implies no easy time, no limitations on the amount of effort necessary to reach the end sought, and no choking off of hard questions, which are a terror to small reputations. The real teacher is ready for anything that will make his work go. He invites hard work. He expects difficult questions—many which he must "look up," many more which he can not answer at all—for he covets *thinking* students. Books—especially laboratory manuals—will help at this point, though they can not meet all cases. Unique disturbing elements will creep in and raise unforseen and unforseeable questions. These are puzzling; sometimes unanswerable. But in reality only

the *little*, narrow-minded bigot can answer all questions. His mental vision is so limited and so obscured by the denseness of his conceit that he is unable to see that 99 per cent of his answers (?) are not answers at all. Personal experience as a student working under sixty-seven college and university men representing eight or ten different institutions puts beyond all question that the broadest, deepest, most acute, most profound and independent men are the very ones who meet most questions that they can not answer and who make a provisional, tentative, or theoretical answer to the largest number of other questions. Not a single strong man in this number could answer one-tenth as many questions, or was one-tenth as sure of those he did answer, as the little two-by-four country professor who kept the first school I ever attended, and who made a personal attack on my father for presuming to give me a solution of a problem in "compound numbers" different from *his solution* (?), and who "licked" me lustily for suggesting that *"ab"* should spell *"abe"* if *"ba"* on the previous page spelled *"bay." He* said it spelled *ăb*, and *therefore* it did. Such is the real status of things—the contrast between ability and imbecility. No teacher worthy of the name will decline to adopt a method which will inspire thought because perchance hard questions may come and despoil an embryonic and quite probably imaginary reputation.

As already implied, laboratory work by the student entails far more work on the teacher. If his supply of apparatus is limited so that he must do much devising and constructing, he may be sure that his course will demand more than twice as much of his time and energy as would a text-book course covering the same ground. The theoretical preparation is somewhat greater, for he must know his *subject* more thoroughly. If he has as many as forty students the reading and criticism of their notes will take fully as much time as his theoretical preparation. Then he must spend twice as much time in the laboratory as he would in the recitation room, while the nervous strain is more intense. But the difference in results will much more than justify the difference in expenditure. And the chief concern of the real teacher is about *results*—not what they cost.

As a last objection, lack of time in the allotment made to

physics in the course of study is urged by some. This, if real, may be serious. A given amount of ground can not be covered by this method in the time required by either of the other methods. One must choose between quantity and quality of work, if the amount of time at his disposal is not sufficient to cover the desired ground by the laboratory method. If the difference in quality were slight and the difference in quantity great, the choice would be easy. But the fact is, that the quality difference is almost infinite, while the quantity loss is about one-fifth to one-fourth of the ground that can be covered by the text-recitation method. This being the fact, is not the choice both easy and imperative? But the most serious aspect of the time question is not the quantitative loss in ground covered. It is the allotment and arrangement of the time. The ordinary recitation is supposed to require of the student three periods—two for preparation and one for recitation. The laboratory method in physics should reverse this—taking two for work in the laboratory and one for writing up the results in a note-book. But the two laboratory periods *must* be consecutive, else the work will fail. It is almost useless to attempt experimental work with less than from one and one-half to two hours of consecutive time. If in the adjustment of the schedule this condition can not be realized, it would unquestionably be better to leave physics out of the course, since laboratory work is then impracticable, and any other kind of work is not only barren of good fruit, but must result in injurious memory cramming plus a total misconception of the nature of physical inquiry and of the actual method of scientific procedure.

TOPICS SOMETIMES NEGLECTED IN TEACHING ELECTRICITY.

BY N. HENRY BLACK.

Instructor in Physics and Chemistry, Roxbury Latin School, Boston.

Do your students ever have difficulty in calculating the specific resistance of copper? My students never fail to have trouble with this experiment. They can find the resistance of twenty meters of No. 30 copper wire by means of the Wheatstone Bridge, and they have already learned by experiment that the resistance of a wire varies directly as the length and inversely as the area of cross-section. But when they are told to calculate the resistance of a cube of copper one centimeter on an edge, they get badly twisted up. For a time I used to blame myself because of some fancied vagueness in my explanations; but recently I have decided that most of the difficulty lies, not in the method of teaching, but in the queerness of the problem.

The standard of specific resistance, *i. e.*, the resistance of a cube one centimeter on an edge, is about as awkward a unit for practical purposes as the genius of the pure physicist could well invent. The trouble is, not because the metric system is employed, but because the centimeter is too short for a unit of length of wire, and the square centimeter is altogether too large for a practical unit of cross-section. If we feel that we must use the metric system in all experiments, why not take as our unit of specific resistance the resistance of a wire one meter long and one millimeter in diameter?

But I am inclined to go a step farther—in order that we may get a little nearer to the actual method used commercially in calculating resistances—and accept as the standard of specific resistance the "mil-foot."[1] The resistance of a wire one foot long and one mil (0.001 inch) in diameter is used as the standard almost universally in this country for commercial purposes. Why not use it in the schoolroom? This will make it easier for us to give problems which have about them the smack of real life, instead of the ridiculous sort we sometimes find in problem books,

[1]See p. 90 in Jackson's *Elementary Electricity and Magnetism* (Macmillan).

where the object seems to be to test the pupils' powers in arithmetic rather than in physics. Then, too, in some of these problems, why not make use of those very handy wire tables which the wire manufacturers get out? These tables give for the various sizes of wire, according to the Brown & Sharpe gauge, the diameter in mils, area in circular mils, pounds per 1,000 feet, pounds per mile, feet per pound, ohms per 1,000 feet, ohms per mile, ohms per pound and feet per ohm for copper wire at 75° F. With these tables at hand, how easily one can solve an apparently difficult problem, and these are usually the sort that turn up after we get out of school. Even if we do need to calculate a resistance not given in the tables, we can use the same general formula, $r = \varrho \frac{l}{s}$, letting ϱ = resistance in ohms of one mil-foot, and l = length of the wire in feet, and s = area of cross-section in circular mils. For a copper wire ϱ is approximately 10.5 ohms.

It will be seen that in advocating the use of the methods of electrical engineers I am not sacrificing one whit of the principle involved; I am simply urging that we teach pupils how to do the thing as it is actually being done in business every day.

Another neglected subject is that of FUSES. I suppose we all show our classes that heat is one of the effects of an electric current; how many of us show the effects of an excessive current, or an "overload," and how it is prevented? Then there are many allied questions: What are three or four of the commonest types of fuses? What is their proper position in an electric lighting circuit, or on a power line? What really happens when the "fuse blows out," and what does it look like? About how large is a piece of five ampere fuse wire? These are some of the questions which I find my students ask me, and why not try to answer them by showing them the "real stuff"?

If there is one topic which deserves our attention more than all others, it is the ELECTRO-MAGNET. In what electric machine is it not employed? Yet are we not, most of us, content with a very elementary and qualitative discussion of this subject? Is it not worth while at least to show the meaning and application of the idea of "ampere-turns"? I believe we shall find in the near future

a rational and simple way of teaching such principles as flux density, lines of force per square inch, permeability and hysteresis.

One of the great difficulties we have to struggle against in teaching electricity is the fact that we can not show the pupil a *Volt*, an *Ampere*, or a *Watt*, units of pressure, current and power of that invisible something we call an electric current. This creates an air of unreality about the whole subject, which can be dispelled only by the students' constantly using the unfamiliar idea or term. The inexpensive direct-reading volt-ammeters now on the market have done more to make the terms volt, ampere and watt mean something to secondary school students than anything devised in the way of apparatus for ten years. But while it is easy to use these instruments with our ordinary battery currents, why should we not have in the laboratory at least one standard voltmeter and ammeter, and teach the proper use of these instruments with commercial currents? It is not enough to say that we connect the ammeter in series and the voltmeter in parallel, as most teachers in the engineering schools will testify.

But someone will complain that I am advocating the training of mechanics rather than the training of boys to think and use the fundamental principles of physics. Far from it. I believe in going to the bottom. In teaching the voltmeter and ammeter, I should use the d'Arsonval galvanometer. In fact, I am strongly of the opinion that we must very soon throw aside our tangent and astatic-needle galvanometers, and make use of the excellent forms of the d'Arsonval galvanometers, which are now on the market. This type of instrument combines sensitiveness, compactness, and "dead-beatness." With a d'Arsonval galvanometer and suitable resistances we can easily show the pupil experimentally the fundamental difference in use between the voltmeter and ammeter.

Another subject which seems to me to deserve more attention than it usually gets in our physics course is that of DYNAMOS and MOTORS. It is difficult for us schoolmasters to realize the tremendous importance of dynamo-electric machinery in the industrial world. Should we, therefore, not give a little more attention to this phase of physics than we did ten years ago? In the first place, it seems to me that we can teach the mutual action of a current-

carrying conductor and a magnetic field in such a way that it shall greatly assist the pupil to see directly the fundamental principle of the dynamo and the motor. One very simple way of looking at this phenomenon is as follows:[2]

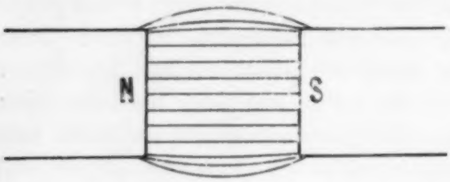


Fig. 1.

Fig. 2.

Let Fig. 1 represent the poles of an electro-magnet. The magnetic field between N and S consists of practically parallel lines of force. Fig. 2 represents the cross-section of a wire and the magnetic field about it when the current flows downward, *i. e.,* into the paper. Now, suppose the conductor carrying the current is placed in the space between the polepieces of the electro-magnet. What is the result? We have the superposition of two magnetic fields—one parallel and the other circular—and the resulting magnetic field can be predicted by the law of parallelogram of forces.

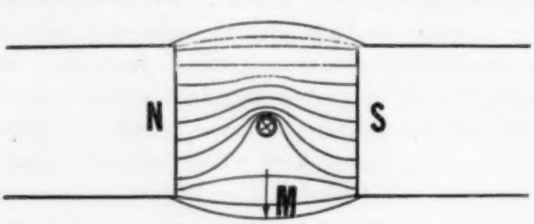


Fig. 3.

Figure 3 shows what we actually get by doing the experiment with iron filings. Moreover, we find that the wire, if free to move, does move in the direction indicated by the arrow M. It moves then as if these lines of force were elastic bands and tended to crowd the conductor at right angles across the field. This, then, is the fundamental phenomenon which is made use of in the electric motor.

[2]See p. 736 in Watson's *Textbook in Physics* (Longmans).

Right here is a good time to teach the use of Fleming's Rule.[3] This helps one to remember the direction of motion of the wire, when we know the direction of the flux and the current in the wire, by the use of the first two fingers and the thumb of the left hand.

Following this, it is only a short step to introduce the principle of the dynamo. We have our same magnetic field and conductor; but in this case the conductor is moved by some mechanical force, and the result is that we have a current induced in the conductor which tends to oppose its motion. This is, of course, the so-called Law of Lenz. For example: In the above case, in Fig. 3, if we move the conductor as indicated by the arrow M, the current induced will be in the opposite direction to that indicated in the figure, *i. e.,* it will be outward from the paper. And right here is a good chance to show the meaning and use of the back electro-motive force in a motor.

Nor would I stop with just these fundamental conceptions of dynamo-electric machinery. Why not show the student that the voltage of dynamo depends on the speed, on the number of turns on the armature, and on the flux density between the pole-pieces? In considering this last point, it would be convenient to discuss the magneto-motive force and the reluctance of the magnetic circuit. These are matters which can be demonstrated in the laboratory or on the lecture table, but we can not expect to do it with the cheap toys we sometimes dignify by the name of apparatus.

Then, too, let us by lantern slides or by actual trips to electric plants, give the pupils a glimpse of the real thing as we see it in the modern industrial world.

It is not going to be many years before we shall have to give a really considerable portion of our time in studying electricity to ALTERNATING CURRENTS. We can already show the principle of the alternator, for all our direct current generators are generating alternating currents which have to be rectified by some commutating device. The application of the alternating currents to electric lighting and principle and use of transformers are well within

[3]See p. 162 in Sheldon's *Dynamo Electric Machinery* (Van Nostrand).

the range of school boys. It is a little harder for the young student to grapple with the problem of the induction motor and the conception of polyphase currents; but that is not so very strange, for these subjects puzzle some of us older heads.

Some years ago I asked a clever electrician, who was engaged in all sorts of electrical construction work, what he thought was the most important principle for the youngster to get hold of in studying electricity, and he said, "OHM'S LAW." Now, I agree with him, if we consider it in all its various forms.

The commonest form which this law takes algebraicly is i (amperes) $= \frac{e \text{ (volts)}}{r \text{ (ohms)}}$. But we also need to emphasize the other forms which this equation takes; namely, $i\ r = e$, and $r = \frac{e}{i}$. Now, e stands not only for the total E. M. F. in a circuit, but also for the difference of potential between two points in a circuit, and r stands for the resistance between the same two points. Let us not confine ourselves in our problems to cases of batteries, but rather include dynamo circuits. Sometimes we entirely forget the internal drop in potential and speak of the terminal voltage as if it were the total E. M. F. Why not show the application of Ohm's Law in the volt-meter method of measuring high resistances and the volt-ammeter method (so commonly used in shops) of measuring rather low resistances?

If a pupil really has grasped the principle of Ohm's Law, why should he not be able to solve a practical problem like the following (which was recently given to a class in electrical engineering, but which ought to be within the power of secondary school students):

"Sixty storage cells, connected in series, supply current to 100 16 c. p. incandescent lamps, of 220 ohms each, connected in parallel. Each cell has a total E. M. F. of 2 volts and internal resistance of 0.002 ohms. What must be the resistance of the leads between battery and lamps in order that there may be a P. D. of 110 volts at the lamps? What is the total current; the P. D. at the terminals of the battery; the power lost by internal resistance; the power lost in the leads; and power consumed by the lamps? Express the result also in foot-pounds per second."

It would be easy to continue this list of what seem to me neglected points in the teaching of electricity, which is based on

my own personal shortcomings; but my purpose is rather to urge that there may be points in electricity, which usually we have not even mentioned, which would be of real interest to pupils, because they see their application outside the schoolhouse.

Finally, I believe the subject thus presented will yield just as much mental discipline as if it were taught in the time-honored stereotyped fashion with apparatus designed to show the makers' skill in making a seven-in-one apparatus, rather than by apparatus built to illustrate a single point so strikingly as to be immediately useful in understanding the electric machinery of today.

APPARATUS FOR DRAWING HARMONIC CURVES.

BY WALTER P. WHITE.

Cornell University.

Most elementary text books of physics, in treating of sound, give diagrams illustrating complex wave forms and showing how these are derived from simpler components. Manifestly, if these diagrams are of value it would be much more valuable for the student to see them drawn and actually built up out of simpler curves. It would be still better if he could draw them himself by means of a simple machine. Such an exercise would not be a substitute for the severe quantitative work of the laboratory, but it might even excel that in arousing interest and in helping the student to gain that *familiarity* with fundamental conceptions which is the most important thing in the knowledge of any subject and oftentimes the hardest to secure.

There are various well known pendulum devices for drawing harmonic curves, but these do not seem to have been very generally used, probably because, while very easy to make, they are somewhat tedious to operate, and give readily only curves composed of motions at right angles to each other (Lissajous' figures), which are very pretty, but do not illustrate the subject as well as wave form curves. An evident improvement is to build up trains of geared wheels and, by connecting rods or some other method, to derive harmonic mo-

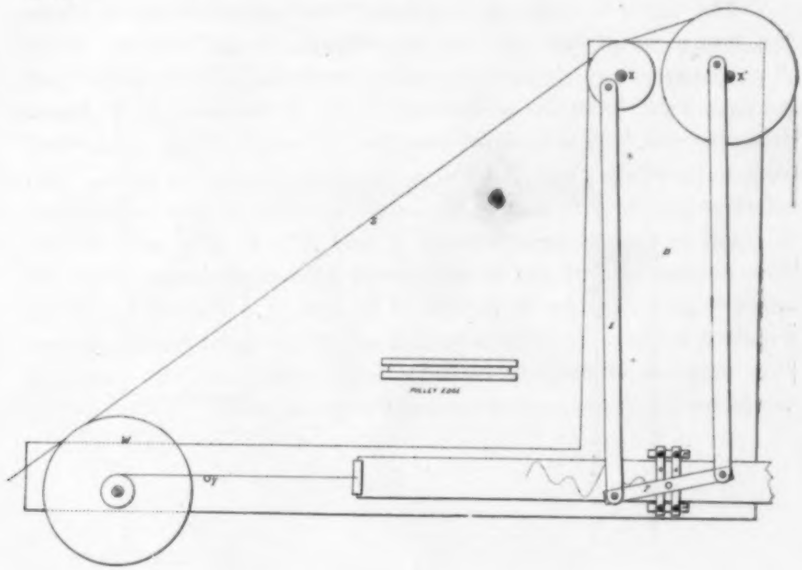
tions from them. It is very easy to devise satisfactory apparatus of this type. There is, however, a method much cheaper and simpler, capable of more rapid adjustment for different curves and at the same time more than accurate enough for all practical purposes. The essential feature of this method consists in using wooden pulleys of different sizes and rotating them all by means of a cord which makes one turn around each. We are thus able to turn them at any rate we please and so to watch the various motions as carefully as we wish. Irregularities in the speed of turning do not affect the curves, since all the pulleys turn together. The same cord draws along the paper in agreement with the circular motions, thus avoiding the difficulty of getting uniform rectilinear motion, which occurs with the pendulum devices.

The planning of a machine on this principle presents no difficulties; nevertheless, a description of one may be useful. The cost may run from nothing to a dollar, according to circumstances.

The form of the pulleys is shown in the figure. The diameter across the bottom of the groove is the only important dimension. A very full set consists of two pulleys, each 8 cm. in diameter, one a little more or less than 8 cm. (for illustrating the phenomenon of "beats"), and one each 4, 12 and 16 cm. in diameter. Beside these there will be needed one pulley of two diameters, like a wheel and axle; the ratio of the diameters should be from 1:6 to 1:10. Each pulley should have a hole through the middle, fitting closely the metal pins on which the pulleys are to turn.

The frame is in the shape of a letter L. The figure shows the whole apparatus arranged for wave form curves. The metal pins, XX^1 (which may be wire nails with the heads filed off), are driven permanently into the frame at the end of the broader arm, *B*, of the frame. On them are put any two pulleys we may wish to use. These pulleys are turned by pulling on the cord, *S*. The connecting rods, *EE*, and the crossbar, *F*, are light wooden rods, pivoted together as shown. Pieces of lead pencil make convenient pins for this purpose. Through the other ends of *EE* run nails which can be driven lightly into the upper surface of the pulleys at any desired distance from the center. Through the middle of *F* projects downward the pencil or other marking device which makes the

curve. This plays between the two strips of wood, *GG*, which serve as guides, compelling it to move in a straight line. Thus, when the pulleys turn, the marker has a motion which, referred to that of the pulleys, is harmonic, compounded of motions due to the two pulleys. The amplitude of either component depends upon the distance from the center of the pulley at which the connecting rod is attached. The periods of vibration vary as the circumferences of the pulleys used.



The harmonic motion thus obtained is drawn out into a curve by moving the paper along at right angles to it, under *GG*, between the paper guides, *HH*. Instead of paper, a strip of blackboard material, rough celluloid, or other substance from which marks can be erased, might be preferred. If the cord, *S*, were attached directly to the paper, a single wave would ordinarily reach nearly the whole length of the paper. The object of the double pulley, *W*, is to shorten the waves by reducing the motion of the paper. The cord, *S*, passes once around the larger portion of the pulley, while another cord is fastened to the smaller part, and pulls the paper along.

The apparatus can be very quickly adjusted for Lissajous' figures. *W* is removed, a pulley is put on the pin, *Y*, another on *X*, and the two connecting rods are joined directly at a right angle by means of the marking pencil; *F* is not used, and the paper remains stationary. Evidently, if the machine is to be used in this way, the distance from *Y* along A to a point opposite *X* should equal the effective length of the connecting rod, and the pins which connect *E* and *F* should be of the same size as the marking pencil.

The figure is drawn to scale and represents a machine where the two arms of the "L" are respectively 26 and 10 cm. wide. *B* measures 60 cm. from the inside corner out. X, X^1, and Y are set each 5 cm. from the nearest side of the frame, and X, X^1 5 cm. from the end. This requires the effective length of the connecting rods to be 60 cm., and *Y* to be as far from the corner as *X*. The effective length of *F*, that is, the length between the two outer holes, is equal to the distance between X and X^1; in this case, 16 cm. The narrow arm of the frame should be enough longer than the other to give room for *W* outside of the pin, *Y*. The total material required is the seven pulleys (which are just as good for demonstration purposes if roughly turned), four metal pins, two pieces of board for the frame, and seven light strips of wood.

THE PREPARATION OF PHOSPHORUS, SODIUM AND POTASSIUM FOR LABORATORY USE.

BY C. D. SNYDER,

Lowell High School, San Francisco, Cal.

Where laboratory classes are large and the laboratory time of the pupil is limited, I have found it desirable to have the elements, phosphorus, sodium, and potassium, on the supply shelves already cut in pieces small enough for immediate use.

To prepare this material in the ordinary way by cutting each piece with a knife, however, consumes a great deal of the laboratory assistants' time.

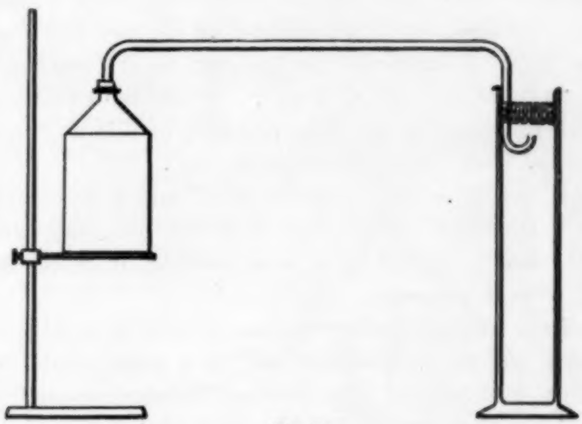


Fig. 1.

A labor and time saving device for the purpose of subdividing these elements into pieces suitable for ordinary laboratory experimentation has been in use in this school for some time; it is so simple and so helpful that I have considered it worth while telling to others.

The general scheme is to melt the substance wanted on a sieve, the mesh of which is the size one wants the pieces of element to be. The sieve is hung in a tall cylinder which is filled with a cold liquid. The liquid should be one with which the element does not react chemically and one whose density is less than that

of the element. The molten globules of the element, passing through the meshes of the sieve, harden in the colder medium below and collect at the bottom of the cylinder as pellets. The accompanying sketch (Fig. 1) will give an idea of the apparatus used.

The can, supported on a ringstand, contains a liquid which vaporizes at a temperature high enough to melt pieces of the element in the sieve at the end of the delivery tube, preferably some of the same liquid with which the cylinder is filled. The can may be heated with a Bunsen burner.

For ordinary phosphorus the cylinder, of course, is filled with water. To keep the oxidation of the phosphorus pellets at a minimum distilled water, from which the air has been expelled by boiling, may be used. Steam generated in the can will soon heat the upper layer of water in the cylinder to the melting point of ordinary phosphorus, which is 44°. Within ten or fifteen minutes enough phosphorous has been prepared in this way to supply a hundred and sixty pupils for a month.

Sodium has a melting point of 95.6° and a specific gravity of 0.9735. Therefore water, even if it did not react with this element chemically, would be a poor medium in which to melt it for the present purpose.

One of the following kerosenes may be used in making sodium or potassium pellets: "Headlight oil" is a water white kerosene with a fire test of 65.5°. The Russian "mineral sperm," "Pyronaphtha," has a fire test of 129.4°. Our own "mineral sperm," sometimes known as "mineral colza oil," has a fire test of 148.8° and a specific gravity of 0.829.

The first of these will be perfectly safe for potassium whose melting point is 62.5°. I have often used it, but with caution, for sodium also.

MEASUREMENT OF WAVE LENGTH OF LIGHT BY HIGH SCHOOL PUPILS.

BY C. F. ADAMS.

Department of Physics and Chemistry, Central High School, Detroit, Mich.

Somewhat more than a year ago I introduced the problem of measuring the wave length of sodium light into our laboratory course in physics. Possibly the results of the work and the method, although not new or novel, may interest the readers of SCHOOL SCIENCE.

Ten sets of apparatus were prepared; and, as the method requires two operators, provision was thus made for a section of twenty students. The chief item of the equipment is of course the grating. The ones we have are photographic copies of a Nobert grating having 3,000 lines to the inch or 1,181 to the centimeter. The price at which such gratings are now sold puts them within the reach of all. For the light a Welsbach burner was taken and screwed to the base of an ordinary Bunsen burner, the wire gauze cap being removed. The top of such a burner fur-

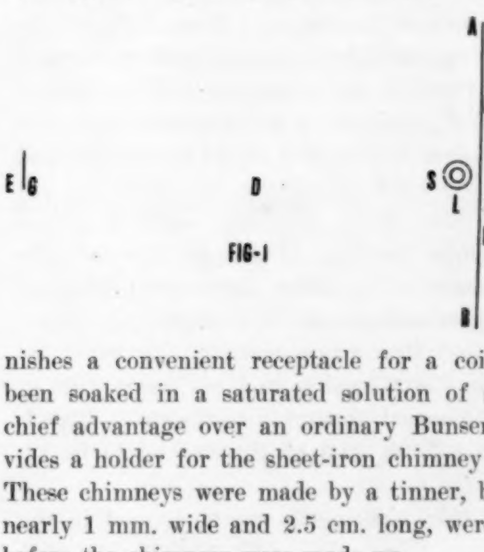


FIG-1

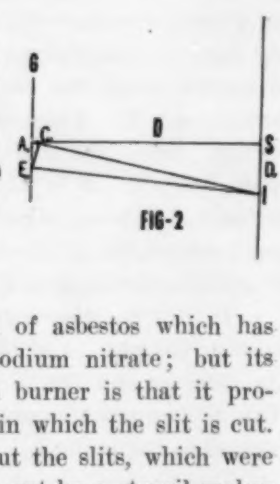


FIG-2

nishes a convenient receptacle for a coil of asbestos which has been soaked in a saturated solution of sodium nitrate; but its chief advantage over an ordinary Bunsen burner is that it provides a holder for the sheet-iron chimney in which the slit is cut. These chimneys were made by a tinner, but the slits, which were nearly 1 mm. wide and 2.5 cm. long, were cut by a stencil maker before the chimneys were made up.

Fig. 1 shows the arrangement of the apparatus. AB is a meter-stick held by a clamp in a horizontal position just back of the lamp L and on a level with the slit s. The grating G is placed from 2 to 4 meters from the meter-stick. Let D represent this distance, which is to be measured by the pupil. An observer on looking through the grating toward the slit s sees two images of it, one at I and one at I'. The position of these images on the meter-stick is fixed by a second operator by cardboard markers which slide along the meter-stick. One-half of the distance $I\,I'$ gives the displacement of the image or d. The secondary images of the slit may also be used and one-fourth of their distance apart will give the distance d. The room must be darkened, but not too much, else the markers can not be seen with sufficient distinctness to be placed accurately.

The grating constant δ being given and the distances D and d being determined the wave length λ can be computed by the formula, $\lambda = \frac{d\,\delta}{D}$.

The theory may be stated very briefly as follows: Let G (Fig. 2) be the grating and the short lines the opaque parts and the spaces the transparent parts of the grating. Then A_1E_1 will be the grating constant δ. Suppose that the light is passing through the grating toward S and that I is the image of the slit. With I as a center strike the arc E_1C_1, then A_1C_1 will represent one wave length or λ. The triangles $A_1C_1E_1$ and A_1SI are similar and $\frac{A_1C_1}{A_1E_1} = \frac{SI}{A_1I}$, or $\frac{\lambda}{\delta} = \frac{d}{D} \therefore \lambda = \frac{d\,\delta}{D}$, in this case $\lambda = \frac{d}{D \cdot 1181}$ cm. the distance A_1S being taken equal to A_1I. If thought desirable, the pupil can calculate the length of A_1I from the known values of D and d and thus this approximation may be avoided.

It will be observed that the images seen by the pupil are virtual, not real; and that Fig. 2 applies to a projection back of the observer and not to the actual apparatus as shown in Fig. 1. This is perhaps the most difficult part of the method to be explained to the student.

During one semester this experiment was performed in our

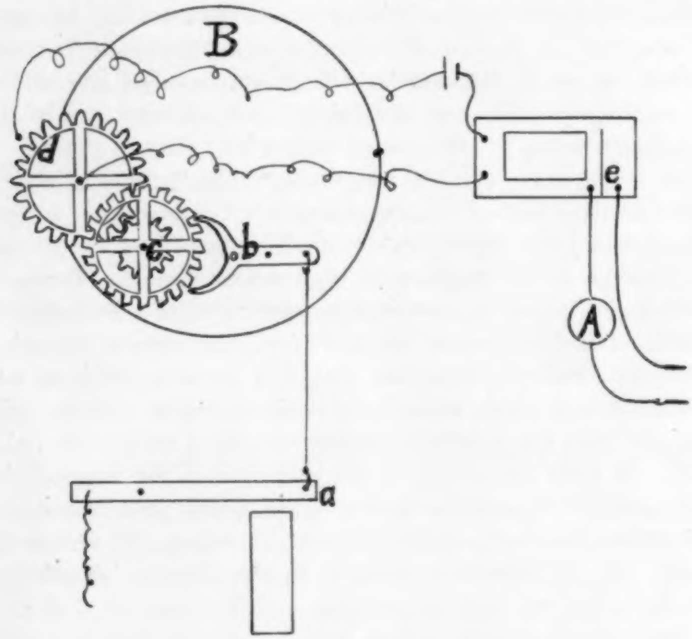
laboratory by forty-three sets of pupils. It should be remembered in considering this report that in a class of nearly ninety pupils there are good, bad and indifferent workers; the work of all is included. The average result for the class for λ was 0.000588 mm., the highest value reported being 0.000600 mm. and the lowest 0.000581 mm. If 0.000589 mm. is taken as the correct value for the wave length of sodium light, I find that the results of all the groups of pupils except one were affected by an error of less than 1.5 per cent; 35 of the groups had errors of less than 1 per cent; 22 groups secured results within 0.5 per cent of the correct value; and 9 groups, within 0.1 per cent. Twelve of the groups reported results above and thirty-one below the correct value. In one or two instances excellent results reported were obtained by averaging numbers in which comparatively large errors offset one another, but on the whole these results fairly represent what can be done with this problem by the average high school class in physics.

It seems to me that the results attainable, the ease of manipulation, and the cost of the apparatus, which aside from the parts already found in every laboratory need not exceed $1.50, commend this exercise to all teachers of high school physics. It may be questioned whether a considerable portion of the students fully apprehend and appreciate the theory of the experiment and the principles involved. Granting that they do not, yet it seems to me that it is desirable somewhere in the course in physics to show the pupil that the wonderful measurements of science are not the results of mere theorizing on the part of men far removed from their world of thought and action, but that they are real and actual and within the reach of the ordinary individual. If for no other reason, this experiment is valuable in the physical laboratory.

A LECTURE EXPERIMENT WITH WIRELESS TELEGRAPHY.

BY V. D. HAWKINS,
Joliet Township High School, Joliet, Ill.

In connection with the lecture on wireless telegraphy it adds much to the interest to perform, besides sending signals, one or two striking experiments. The apparatus shown in Figure 1 is compact, simple and inexpensive. By its use we turn on or off an electric light, start and stop a motor, or start and stop a bell.



B is an ordinary alarm clock obtained from a second-hand dealer at a cost of twenty-five cents. The balance wheel controlling the escapement *b* has been removed. A wire connects *b* with the telegraph sounder *a*. When the signal is received by the wireless telegraph instrument the sounder *a* moves down and up carrying *b* with it. This advances *c* one cog and *d* one-half a cog.

This makes a contact at d and the Morse telegraph relay at e connects A, which is either a light or a motor.

Whenever the lecturer wishes to stop the motor a second signal advances d one-half cog, the contact is broken at d and the motor stops.

DEVICES FOR PROJECTION WITH THE MICROSCOPE.

BY E. R. DOWNING.

Department of Biology, Northern State Normal, Marquette, Mich.

Some teachers may have in their equipment both the stereopticon and compound microscope without realizing that they can be used together advantageously. Set the microscope on a block so the mirror of the microscope is on a level with the lantern objective. Remove this objective and bring the microscope up so the mirror faces the opening made by the removal. Adjust the mirror so as to illuminate the object on the stage. Stand an apparatus support, or ring stand, beside the vertical microscope. By means of a clamp fasten a small mirror, as a flat microscope mirror or hand glass, over the eyepiece at such an angle as to throw the image on to the screen. Both lantern and microscope may be placed on a table some ten feet from the screen, and good views of living animals obtained, such as cyclops, paramoecium, etc. Sections of stems and other plant structures, histological material and crystallizations make equally brilliant showing. As the microscope stage is level, liquids are readily handled on it. A water tank, to absorb heat, is advisable between the light and the microscope, but this may be dispensed with, usually, by avoiding too sharp a focus of light and heat on the object on the stage. A black cloth screen placed at the end of the table farthest from the lantern, extending up to the microscope ocular, will shield the screen from light escaping from the lantern.

Microscopic projection may be successfully tried with only the compound microscope. The room must be darkened, though not necessarily totally dark. Fit a 12-inch board under the lower sash. Stand the microscope with the tube horizontal on the out-

side sill. Cut a hole in the board just large enough to admit the ocular end of the microscope. Let this project through the opening into the room. Make another hole in the board large enough to put your arm through and at such a distance from the ocular opening as to make a comfortable reach to the stage and mirror. The arm when placed in this hole effectually excludes light, especially if part of an old coat sleeve be tacked into the opening so the arm can be shoved through it. With the hand outside one adjusts the mirror to throw sunlight on to the object on the stage, the image of which projected through the microscope into the room falls on the screen. A flat bottle of distilled water strapped to the under side of the microscope stage with rubber bands will absorb the heat rays. By either method low power objectives (2 in.-½ in.) and eyepieces (1-2 in.) with fairly distant screens give better results than high powers and near screens.

Health in Japan.—Health officers say that the death rate for children is lower in Japan than it is in Europe and America. This is as it should be in a country where the houses are off the ground a foot or two, and have no cellars, and the air inside is as fresh as it is out; where, too, in such places at least as Tokio, everyone bathes and has a good scrubbing every day. From 800,000 to 1,000,000 persons go to the public baths of the capital daily, and there are tens of thousands of private baths besides. That is a good showing for a city with a population of less than 2,000,000. —*American Medicine.*

The Modern Crusade Against Consumption is the title of a most enlightening article in the *Outlook* for November 21, by Irving Fisher, one who has been cured of tuberculosis. Every teacher of hygiene should read it.

F. W. B.

Metrology.*

THE PUBLIC LANDS EXCEPTION.

BY SAMUEL S. GREELEY, C. E.

Reasons why "The Completion of the Survey of the Public Lands" should not be excepted from the operation of any bill which may be passed for the adoption of the metric system of weights and measures in the business of the United States.

In the various bills which have been introduced into Congress within the past eight years, providing for the use of the metric system in this country, the survey of the public lands has been specifically excepted. It seems to many persons that this has been done without sufficient reason and in partial ignorance of the facts.

In all the original surveys for the subdivision of the public lands into townships, and of these townships into sections, the Gunter's chain of sixty-six feet, with its decimals, is the unit. Each full section is, theoretically, eighty chains, or one mile, square. A full township is, theoretically, a square of thirty-six such sections. So long as these lands remain unsold, or are held by the purchasers in large tracts (in the original package, so to speak), it matters little in what terms their dimensions may be expressed.

But as population gathers at certain points, and the land is required for human habitation, towns and villages spring up. The sections must be subdivided into lots of convenient size for homes. Roads must be made, sewers and pipes for water and gas must be laid, and wires for electricity must be strung. On the lots houses must be built. All the materials for these various purposes—lumber, masonry, pipes, plaster, paint, glass, wire—are measured by the foot or its multiple, the yard. The architect must therefore draw his plans to a scale of feet, and he requires the surveyor who lays out the lot to give him its dimensions in feet

* Communications for the Department of Meteorology should be sent to Rufus P. Williams, North Cambridge, Mass.

and decimals thereof, or in feet and inches. The surveyor, who subdivides a section or a part thereof into house lots, anticipates all this, and measures its dimensions in feet, and marks upon his plat of the new village or town the size of the lots and the width of streets in feet. So, for that particular tract the original dimensions, given by the United States in chains, pass out of use. When the metric units come into general use all materials will be bought and sold by these units. Architects will draw their plans to a metric scale, and the surveyor will measure the sections by a metric chain, and will give metric dimensions to his lots and streets.

If the survey of public lands be excepted from the operation of a metric law, the United States deputy surveyors will continue to use the almost obsolete Gunter's chain, and will record the dimensions in terms of the chain. In order to adapt the land to use for towns and villages, the land must be remeasured by a metric chain, and all the United States dimensions must be translated into terms of meters. Would it not be a great saving of time and labor if the original survey of the land by the United States were made in the same units, which are to be applied later when the land becomes closely occupied? If the metric system is to be adopted for all other uses, should it not, *a fortiori,* be applied to land which is the basis and the foundation of all things?

The absurdities of our customary system of metrology and the advantages of the metric system have been abundantly set forth in the able reports of Congressional committees, and by many able writers; I need not rehearse them. I only urge that, if the metric system is to be brought into general use, public convenience demands that the survey of the public lands should not be excepted.

No doubt a strong objection to a change of unit from the Gunter's chain to the meter will be found in the "jogs" or breaks of continuity that will occur on the lines where the new method joins the old.

The nearest approach to the size of the present section of eighty chains would be the section whose sides are 1,600 meters long (5,249.4 feet). This is the section that is most likely to be adopted under the metric system. This is 30.6 feet less than the

side of the present normal section. Starting at the south line of a township in the old survey, the jog would increase with each section till, at the north line of the township, the jog would be six times 30.6 feet, or 183.6 feet; and it would increase in like ratio with each succeeding township.

This inconvenience would be felt only on the lines where the two systems meet. It might prove a reasonable, though not a fatal, objection, but for the imperfections and irregularities of the survey of public lands as actually made.

It is a matter of common knowledge among surveyors and land owners that the length of section lines in the earlier surveys, measured by modern methods and with the appliances of today, largely exceeds the length of the same lines as stated on the plats of the original survey. For instance, in T. 39 N., R. 13 E., adjoining Chicago, the average of seventeen east and west lines, carefully remeasured, exceeds the normal length of eighty chains by sixty-four feet. The average length of nineteen north and south line exceeds the normal length by forty-six feet. And similar, or greater, discrepancies are common throughout.

There can be no doubt that the public land surveys, as now executed, are far more carefully and skilfully done than were those of eighty years ago. The "Instructions of 1902," issued by the United States Commissioner of Public Lands to deputy surveyors, are ample evidence of that.

The ever present cause of irregularities and "jogs" in both old and new surveys is the convergence of meridians. Under existing laws, passed many years ago, townships must be figures bounded by meridians and parallels of latitude, and must be six miles square—a manifest impossibility. In order to meet and minimize this difficulty the instructions of the surveyor-general to his deputies provide that at every fourth township *i. e.*, every twenty-four miles, a parallel of latitude shall be run as a "correction line," upon which the tier of townships thence north shall be laid out the full width of six miles.

In Cook County, Illinois, such a correction line was run in the original survey of seventy years ago as the boundary between townships 41 and 42 north. On the north line of Hanover, T.

41 N., R. 9 E., for example, the jog, or offset, to the section lines running north vary from fourteen chains to 24.50 chains—or from 924 feet to 1,617 feet. This is from five to nine times the maximum jog in one township, caused by the difference between the present section and the metric section. These irregularities, caused by the spherical form of the earth, will continue, whatever standard of measurement be used. They will not be diminished by the continued use of the chain; they will not be increased by the use of the meter.

The meter and the hectare share with the Gunter's chain and its acre the great advantage of being decimal. The meter has the vastly greater advantage that, when adopted, it will be the common unit of measurement of all objects bought and sold by dimension. The farm, the garden, the house, and all that goes to the making of the house, will be measured and estimated by the one common standard. Nobody measures, or ever will measure, anything but land with a Gunter's chain. That instrument has had its day of usefulness and has served its purpose well. Now let it make way for a successor better fitted to serve present needs.

Nobody ever measured, or ever will measure, anything but land with a Gunter's chain, and it is fast passing out of use for that purpose. All surveys for railroads, canals, irrigation, and other public works, are made, not in chains and links, but in feet and decimals. After a continuous practice of fifty years as a land surveyor in Chicago and Cook County, I may say that I have not used, or even seen, a Gunter's chair for forty years. That instrument has had its day of usefulness, and has served its purpose well; now let it make way for a successor better fitted to answer present needs.

It may be urged that only a comparatively small part of the total area of the public domain is likely to be occupied by cities and villages. Much of it will be used for farming, grazing, forest, mines, etc., and vast tracts of mountain and desert land will remain long, perhaps forever, unoccupied. But even these vacant and apparently waste tracts can not escape the activities of man. They are part of a great whole, and the conditions of the whole must be studied and known, for the benefit of each part. Physical

conditions, geography and topography, must be learned; elevations and contours must be taken, rainfall determined, and other important problems must be solved.

When the metric system is adopted for all public uses, except for the survey of public lands, the civic and mining engineer, the meteorologist, the geologist, will make and record their measurements and observations in terms of the metre. Then the chains and links of the United States deputy surveyor must be reduced to the metric standard, just as in the case of land which is to be covered by a city or a village, and the same unnecessary labor must be spent.

This Government has lately bought the lands of the Friars in the Philippines. It is probable that there, as in other islands acquired from Spain, will be great public domains, for which some system of survey must be devised. Spain is a metric country; what better system could be found than the metric, which is already more or less familiar to people, who have lived under Spanish influence?

THE METRIC SYSTEM PSYCHOLOGICALLY CONSIDERED.

BY WILLIAM F. WHITE.

(*Continued from page 459.*)

It is of importance to the mind that all the denominations of any table be related to one another by the easiest possible scale. As the notation of numbers among all civilized peoples today is decimal, and seems certain to remain so, the decimal scale of weights and measures is the only one that can be defended psychologically.

It is also important that the units of different tables be related to one another in the easiest manner. The metric system alone meets this requirement.

Contrast the mental effort necessary to determine what part of a rod is five feet, with the ease of determining what part of a hektometer is five meters. The latter is seen at a glance to be

five hundredths. Again, compare the labor involved in finding how many cubic inches in three quarts, with that in finding how many cubic centimeters in three liters. As before, the answer to the metric question is almost instantaneous. To find the number of cubic inches in a given number of quarts, one must use a number that is awkward as a factor and difficult to remember; not to mention that he must be told whether the quarts are liquid or dry.

All unnecessary mental labor in reduction uses up the energy of brain cells to no purpose but withdraws that energy from productive channels.

The convenience of the system of standard time which we enjoy should furnish a suggestion of the convenience of a rational and universal system of weights and measures.

Why should the nation that first adopted decimal coinage—a step so manifestly advantageous that it has since been taken by most of the progressive nations—now hesitate to adopt decimal weights and measures? Any one knows that the most ignorant immigrant to our shores, whether from a country possessing a decimal coinage or not, has no trouble with ours. The fact that there are 100 cents in a dollar, is mastered with marvelous celerity. In the metric system our weights and measures will be decimal also.

How much time would be saved in the school life of every boy and girl if the present customary weights and measures were entirely replaced by the metric? The writer received replies to this question about three years ago from more than a hundred fifty teachers in New York state and from many state and city superintendents of education. Several superintendents replied that they had not sufficient data for an estimate. Many of the replies were so worded as not to be easily tabulated. The mean of all the estimates received would place the time saved at somewhat less than a year. Others, however, who have had opportunity for more extended inquiry have almost uniformly reached the conclusion that the time saved would be a year or more. In 1877 Dr. B. G. Northrop, secretary of the Connecticut board of education, wrote that American teachers generally concur in the opinion that at least one year in the school life of every pupil would

be saved.[7] At about the same time a circular letter issued by twenty-six American educators said: "Careful computation of the result of completely replacing the present weights and measures in our arithmetics by the metric gives a saving of a full year in the school life of every child educated." To about this conclusion committees of the British parliament and the United States congress have recently come. A circular issued by the president and secretary of the American Metrological Society and afterward made part of the report of the committee on coinage, weights, and measures of the house of representatives, submitted March 1, 1901, says: "Conservative educators have estimated that the use of the metric instead of the customary system of weights and measures would result in a saving of from one to two years of the school life of every child. The already overcrowded curriculum of the public schools makes this a matter of the highest importance, and so great an economy would alone justify the adoption of the decimal system."

HISTORY, EXTENT OF USE, AND PROSPECTS OF METRIC SYSTEM.

History knows no time when men did not use weights and measures. In all lands the weights and measures of primitive men were very crude, and there was great variety in the ratios of one denomination to another. Among the various national systems of historic times the numbers most frequently occurring as ratios are 2, 3, 4, 5, 10 and 12; but there are many others, some of them exceedingly inconvenient numbers (e. g. $1\,{}^2/_3$, 7, $7\frac{1}{2}$). Perhaps some trifling circumstance caused their first use and they continued to be employed through habit, there being no system. When it is remembered how widespread was the tendency, even among the earliest men that have left records, to count by ten, one is inclined to wonder that ten does not appear oftener in the scales of denominate numbers. The probable explanation is that primitive man could not do without concrete things as counters. For numeration he counted on his fingers—hence by ten; while in measuring he used concrete units—in measures of length usually parts of the body for short measures, as foot, cubit, ell, and for

[7]Northrop, B. G. *Lessons from European schools & the American centennial*, N. Y. [1877] p. 24.

longer measures a reed of average length or other rough natural unit that might be convenient—these concrete units sustaining to one another such relations as might happen, or being altered slightly to make the ratio a whole number. In subsequent measurements the mind would naturally follow the path that had once been taken. Thus there would grow up a table of measure. Changes were doubtless in the direction of simplification as the advantage of an easy scale came to be appreciated; but the need was not felt for a system with a uniform scale corresponding to the radix of notation of numbers. Japan, however, always had a decimal system. Since 1885, when Japan became a party to the international metric convention, her customary weights and measures have been assimilated to the metric units. China's system also is decimal (though not metric). But China and Japan did not have much influence on the world's civilization.

The similarity in names of units (e. g. the *foot* occurring everywhere) in the metrology of Europe down to the metric age, points to a common origin—Rome. But there was great diversity in the ratio of unit to unit and in the size of units bearing the same name.

(*To be continued.*)

NOTES.

The New Metric Bill. Hon. John F. Shafroth, of Colorado—father of the recent metric bills—introduced into the House of Representatives, November 9, 1903, the first day of the first (extraordinary) session of the Fifty-eighth Congress, H. R. Bill No. 93, as follows: "Be it enacted," etc., "That on and after the first day of January, 1905, all the departments of the Government of the United States, in the transaction of all business requiring the use of weight and measurement, except in completing the survey of public lands, shall employ and use only the weights and measures of the metric system; and on and after the first day of January, 1906, the weights and measures of the metric system shall be the legal standard weights and measures of and in the United States." It was referred to the Committee on Coinage, Weights and Measures, which consists of the following eighteen members: James H. Southard, Ohio; Justin D. Bowersock, Kansas; Thomas Hedge, Iowa; Arthur L. Bates, Pennsylvania; George W.

Cromer, Indiana; John W. Dwight, New York; William M. Lanning, New Jersey; James W. Brown, Pennsylvania; Solomon R. Dresser, Pennsylvania; George J. Smith, New York; Charles F. Cochran, Missouri; John F. Shafroth, Colorado; James M. Griggs, Georgia; John W. Gaines, Tennessee; Ezekiel S. Chandler Jr., Mississippi; John S. Rhea, Kentucky; Timothy D. Sullivan, New York; J. S. Wilson, Arizona.

Recommendation from High Authority. Secretary of the Treasury Leslie M. Shaw, in his Report to the Speaker of the House of Representatives for the year ended June 30, 1903, made the following significant and urgent recommendation:

"During the year the attention of this department has been forcibly called to the growing need for international uniformity in so fundamental a necessity to trade as weights and measures. The Customs Congress of American Republics, held at New York, strongly urged the adoption of the metric system to simplify the transaction of government business in connection with international trade. Moreover, the National Board of Trade of the United States, the Board of Trade of Canada, and the Congress of Chambers of Commerce of the British Empire have recently urged by strong resolutions the adoption of the metric system. The experience of forty countries of the world has proved beyond question that the international metric system is unsurpassed for practical convenience, possessing as it does a terminology concise, definite and free from ambiguity, affording a maximum facility in the countless transactions and computations of science, manufacture and commerce."

R. P. W.

Book Reviews.

Bacteria, Yeast and Molds in the Home. By H. W. Conn, Ph. D. 293 pages. Ginn & Co., Boston, 1903.

In this volume we find a radical departure from the ordinary book having instruction as the main feature, in that the work is intended primarily for the housewife or other person in charge of the home and not as a school book. The subject matter comprises the interesting forms of molds, yeasts and bacteria that have so much to do with human welfare, either as affecting the nature and permanency of the food supply or as causative of disease or unsanitary conditions.

The subject matter is presented in accurately scientific and yet eminently readable form, and with such intelligible statements that the average housewife may be able to understand and utilize the information given.

Brief descriptions are given of the many forms treated of and well chosen illustrations add to the value of this part of the book. The rela-

tion of the various plants to human economies is clearly and fully pointed out and proper methods of prevention of growth, or destruction, if present, increase markedly the practical value. This is particularly to be commended in the remarks about smallpox or diphtheria.

It is a good book for any woman to have and become familiar with, or, for that matter, for any person, man or woman, who has at heart the best welfare of the home.

H. S. P.

A Laboratory Manual for Physical Geography. By FRANK W. DARLING, Head of Department of Geography, Chicago Normal School, and RALPH E. BLOUNT, JANE PERRY COOK, KENNETH C. FITCH and CALVIN L. WALTON, Instructors in Physical Geography, Chicago High Schools. Two Parts. Atkinson & Mentzer, Chicago & Boston, 1903. 50 cents.

For several years it has been plain that a system of practical exercises in geography must be developed. This was necessary to place the subject on a par with other sciences as an effective discipline and especially if the subject were to claim standing in college entrance. This manual is one of the earliest and best attempts to put geography on such a concrete basis, and it marks at the same time, the progressive energy of the teachers who have joined hands to produce it. There was appropriateness in this joint work, because all the exercises which are offered have been in actual use in the laboratory.

The manual is in two parts, of which the first is devoted to directions to teachers. In this the construction of simple apparatus is given appropriate place, and directions are given for furnishing and arranging a laboratory. These directions are made plain by a variety of good figures. Plans are given for the use of water in dynamic experiments by each pupil, a feature sure to become important as geographic education develops. Another valuable feature is the list of apparatus with estimated cost of many of the items. The list is in two parts, one giving a smaller outfit, and the other a complete laboratory equipment.

The second part of the manual gives the exercises for the student. This is in flexible board covers, with 8x10 inch detachable leaves, perforated. Thirty-two exercises are offered. Some of these are quite extended, so that there need be no doubt that the material is adequate to a good and substantial course in the subject. The first thirteen exercises are occupied with the study of the atmosphere and with subjects commonly classed under the head of mathematical geography. The remainder are upon the lands, a deservedly large proportion, dealing with this most important of the sub-topics of the subject. Here we have lessons on minerals, rocks and soils. Land form begins with an exercise in contours, and then we have river development, the Mississippi River, with data for profiles of this and other rivers; glacial topography, glacial materials, shore lines, plains, plateaus, etc. A great merit of this manual is its precise and concrete character. It is based on definite

material and the questions do not allow the student to escape into a maze of generalities.

Professor Darling and his associates have made a worthy contribution to educational geography, while they wisely and modestly recognize that our subject is in a formative stage, and therefore express their purpose to revise the manual from time to time, as the growth of the subject may demand.

Colgate University. ALBERT PERRY BRIGHAM.

Reports of Meetings.

KANSAS STATE TEACHERS' ASSOCIATION—SCIENCE SECTION.

The first regular meeting of the section was held at Topeka on December 30 and 31, 1903. The section was organized in 1902, largely through the efforts of Mr. F. B. Isely, of Wichita, who was elected chairman for the first year.

Prof. S. J. Hunter, of the University of Kansas, in answer to the question, "What training in zoölogy may we reasonably expect of the high school?" gave two points:

First—The faculty of acquiring knowledge through digested thought suggested by the study of natural phenomena.

Second—The formation of habits of accurate observation.

In discussing the paper, Prof. L. C. Wooster, of the Kansas State Normal School, advocated the introduction of nature study in the grades. He called attention to the collection and classification of insects as a means of arousing and holding interest.

Mr. C. E. Houdyshel, of Lecompton, in discussing laboratory work in zoölogy, emphasized the study of local forms. Marine material is all right, but for an inland section of the country local forms are more interesting. The high school foundation for advanced work in this way or any other science should be a broad one. Laboratory work in zoölogy should include the study of morphology, natural history and physiology.

He would begin work with a multicellular form, and thought that the grasshopper or crayfish could be used to the best advantage. When the course begins in the fall of the year, insects may first be studied with profit, as the material is abundant. Laboratory work is not to be condensed into a single part of the course, but should continue throughout the course.

Laboratory work in chemistry was discussed by Mr. C. S. Baker, formerly of Minneapolis, Minn. He called attention to the need of individual work and said that the work done should be very largely quantitative in its nature.

Mr. W. L. Enfield, of the High School, Wichita, discussed laboratory work in physical geography. He advocated the more general adoption of the laboratory method of teaching this subject. There is not the need for such expensive apparatus as is used in the other sciences. All that is necessary for field work is the power to see and the faculty of relating cause and effect. He called attention to several sources of supplies for class-room and inside laboratory work.

Mr. C. H. Nowlin, of the Central High School, Kansas City, Mo., discussed laboratory work in physiology. He gave a list of experiments which might be performed by the pupil, many of them while pupil was at home. He named several medical colleges where it was possible to get slides for microscopic use at a minimum of expense.

The second day's session was taken up with a discussion of "How and What Shall We Teach in Botany?" Mr. J. E. Courtright, of the High School, Beloit, advocated the study of morphology. Study of both seeds and seedlings gives the pupil a basis for the later divisions into classes. It gives a scheme of classification throughout and a complete history of plant life.

Mr. C. E. Johnson, of the Summer County High School, would have the pupil pay special attention to ecology. It should not be divorced from the other phases of the subject, however, but the study of all should go hand in hand. The study of the ecological phase should, wherever possible, take place in the field, reinforced by a critical microscopic study of the structure.

Mr. S. F. Poole, of Fairmount College, Wichita, in discussing the question emphasized the following points: "The student should get the idea that plants are living, working individuals, capable of responding to and adapting themselves to internal and external conditions. Physiology must not be separated from morphology and ecology. Teach it experimentally, letting the students do the work as far as possible." He called attention to the "Standard Option" presented by the Committee of the Society for Plant Morphology and Physiology and recommended the use of its outline.

In discussing taxonomy, Mr. E. S. Weatherby, of the High School, Newton, recommended the preparation of a herbarium in connection with the field work in ecology. There is danger, however, of the study of taxonomy degenerating into mere mechanical drudgery. It also is of value from the point of view of its cultural side, as it teaches neatness, skill of arrangement and cleanliness. Finally, not taxonomy versus physiology, ecology and morphology, but taxonomy with physiology, ecology and morphology.

After electing Cecil E. Houdyshel chairman for the ensuing year, the section adjourned to meet in December, 1904.

Reported by W. L. Enfield.

CENTRAL ASSOCIATION OF SCIENCE AND MATHEMATICS TEACHERS—BIOLOGY SECTION.

The meeting was called to order by Chairman F. B. Maxwell at 2:30 p. m., when the following program was presented:

General Subject: Essentials of a High School Course in Biology.

1. Is the course for college entrance requirements best for those who go no further?
   Prof. J. G. Needham, Lake Forest College (see page 483).
   Prof. Otis W. Caldwell, Eastern Illinois State Normal School. (Professor Caldwell's address will appear in April number.)
2. Discussion of the College entrance requirements in Botany of the Middle States and Maryland Association of Schools and Colleges.
   Mr. William Crocker, Northern Illinois State Normal School.
   Mr. T. J. Headley, Rensselaer (Ind.) High School.
   Mr. W. W. Whitney, South Chicago High School.
3. Current Events.
   Mr. F. Colby Lucas, Englewood High School.

Mr. Crocker was unable to be present, but sent his paper, and it was read by the Secretary. (It is given below.) After the formal papers a number took part in the discussion. The general trend of the discussions and papers seemed to indicate (1) that the course of those going to college and those who do not should be identical and (2) that the amount of work outlined by the Middle States and Maryland Association was too much.

Acting upon the communication of Dr. Cowles, the section instructed the chairman to appoint a committee to act with a similar committee appointed at the Annual Conference of Secondary Schools and the University of Chicago.

The officers elected for the ensuing year were as follows:

Mr. Otis W. Caldwell, Charleston, Ill., chairman; Mr. H. E. Walter, Chicago, vice-chairman; Miss Elma Chandler, Elgin, Ill., secretary.

Reported by FREDERIC C. LUCAS, *Secy.*

DISCUSSION OF THE COLLEGE ENTRANCE REQUIREMENTS IN BOTANY OF THE MIDDLE STATES AND MARYLAND ASSOCIATION OF SCHOOLS AND COLLEGES.

In determining the material and method of a preparatory course in any subject there are two distinct classes of students to take into consideration. By far the larger class includes those whose school life will end with the high school or academy, and the other those whose school work will be continued in the college or university. It is impracticable to plan different courses for the two classes of students. Hence the courses given in any subject must be such as will meet the needs of the two classes to the fullest possible extent—that is, prepare for college and for life work.

The college entrance requirements determine to a considerable extent

what the preparatory courses are, both as to material and method. Since the courses affect a far greater number that do not enter college than the number that do, the entrance requirements must not ignore the question of what is best for those that never enter college.

Is not, however, the best preparation for college also the best preparation for other phases of life? If this be the case, what is the best material and method in a course in botany to meet the two demands?

A number of instructors of botany in universities have urged the following weak points against the students that enter their institutions from even well equipped high schools: "They know very little about the plant life around them; are unable to interpret what they see either in nature or in the laboratory; seem to have little power of drawing conclusions on the basis of evidence, and have their interest in the subject dulled by the courses of the high school."

In short, the students lack just the knowledge and ability that would result from a scientifically conducted nature study course. Hence it has been suggested by some prominent biologists that botany (as well as other biological subjects) be taught as nature study below the college. We must not interpret nature study to be identical with the old phrase "observation work," which emphasized the seeing with little attention to interpretation. By teaching botany as nature study these educators mean that problems concerning plant life, activities or structures shall be put to the pupils by the teacher (or, still better, by nature herself), and that the pupils shall find in plants themselves (or in other portions of nature) the answers to the problems. Here the activity of the child is to a considerable extent similar to that of the scientist who originally worked out the problem and is certainly as great in amount and variety as could be obtained in any way. It is maintained that the morphological courses which have prevailed in high schools until very recently do not call forth the desired amount and variety of mental activity.

Exponents of nature study versus science courses in botany maintain that the following good results will accrue from presenting the work by the former method: The students will have a live interest in plant life, or, as one of the advocates has said, "It will keep the tentacles of inquiry functional"; they will be familiar with plants about them as to name, activity and habit; they will acquire scientific habits of thinking, for all their mental activities will be called into play—the conclusions must be drawn and the evidence to establish them must be mustered by the pupils themselves from their own experiments and observations.

Scientific habits of thinking, live interest in the local plant life, with some insight into it, it is urged, furnishes at the same time the qualifications for good college work in botany and the best preparation for a life that shall be awake to its surroundings and intelligent and safe in its production.

It is probably true that these thinkers do not care so much what is taught as the method by which it is taught. It is true, however, that the

nature study method will demand the curtailing of certain botanical phases to the emphasis of others. Such a course could hardly lead to a complete morphological arrangement of the plant world with the homologies that result from such a course and that are so significant in establishing evolutionary and genetic relationships. On the other hand, it will tend to the emphasis of the physiological and ecological phases of the subject and to the study of structures in the light of function.

In a series of talks before the round-table of superintendents of northern Illinois Prof. John M. Coulter suggested the material and method of several nature study topics in botany suitable for the upper grades or high school. Among others were the following general topics: Leaf coloration, leaf fall, ascent of sap, shedding rain, transportation of seeds in dirt, effect of drouth, relation of light to the green color and growth of parts, position of leaves in reference to light. Each of these general problems breaks itself up into subproblems, the answers to which, as well as the answer to the main problem, are to be found by the children through experiment and observation.

From this brief statement of the stand of the nature study faction let us turn to the examination of the course to be discussed. The course is based on the report of the Committee on Botany of the Science Department of the N. E. A., modified by a committee of the Society for Plant Morphology and Physiology. The committee states the nature and aim of the course thus:

"The following course is designed to include those topics in the leading divisions of the subject which are now regarded by most teachers as fundamental. The general sequence of topics is that recommended, but this point is not regarded as especially important, and the sequence, the methods and the textbooks are left to the judgment of the individual teacher.

"Individual laboratory work by the students is essential and should receive at least double the amount of time given to recitation. Records of the laboratory work, in which stress is laid upon diagramatically accurate drawing and precise expressive description, must be handed in at the time of examination and will count one-third toward admission. The full year's course consists of two parts."

Both the course under discussion and the report of the Botany Committee of the Science Department of the N. E. A. (1899), on which this course is based, were an outgrowth of a defective relation existing between the colleges and universities on the one hand and the high schools on the other: The colleges and universities were slow, in fact, in many cases refused, to accept the work done in science in the high schools as credits for admission. Students entering these higher institutions were required to take the courses offered for beginners in a given science, regardless of whether they had had previous high school preparation in that subject or not.

One method by which these defects could be overcome (and it is the

method suggested by the course under discussion) was to make the botany courses of the high school similar to the minor courses offered in the colleges and universities. A course of this kind leads to a comprehensive view of the subject in morphology as well as ecology and physiology. Such a movement, it is urged, has serious dangers. Botany is generally taught in the freshman or at least in the sophomore year of the high school. Botanical knowledge that freshmen and sophomores of the university, with their more mature and better trained minds, would grasp readily might be meaningless, foreign and insipid to freshmen and sophomores of the high school if presented by a similar method.

It should be cited in this connection that structures have two different phases of significance: the one coming from the functions which they perform and the other from homologies which they establish. The immature mind quite readily grasps the meaning of structures from the functional side, but grasps far less readily their significance from the side of common origin and homology. The course under discussion urges that structures be studied in connection with their functions, but it also makes morphology in which homologies are to be especially emphasized the backbone of the course. Of the twenty-eight general topics to be worked out, sixteen are purely morphological in their nature and twelve physiological and ecological. If we may judge from the explanation in connection with each topic, the morphological phase receives even much more than the sixteen twenty-eighths of the time devoted to the course.

Any one who teaches an elementary course in botany, especially if he be led into emphasizing its morphological phases, feels a growing necessity that the pupil should get with exactness a comprehensive and detailed insight into morphological relationships. This felt necessity on the part of the teacher is an outcome of the essential organization of the subject matter and not of the way in which the child mind grasps botanical knowledge. In reviewing this course one feels that it manifests such a felt necessity. Yet there is perhaps a reason for the preponderance of the morphological phase in this course. The course is to cover a full year's work. The first half of the course, treating of the essentials of botany, studies structures largely under the light of function and adaptation. The last half of the course, which in many cases will perhaps be elective, treats the untouched phase of the subject—classification. It may be asked: Why, after the foundations of the other phases of the subject have been laid, should not the morphological relationships that underlie classification be brought out in some detail?

The course in botany in the Horace Mann School of Columbia University deserves special mention as a course that makes ecology the central idea and studies structures in this perspective even in the portion devoted to classification. Lloyd himself says of the course: "It will be observed, however, that the ecological point of view is maintained throughout, for it is contended that the small amount of work which the curriculum permits

is not sufficient to lead the student to any but a very imperfect conception of the relationships of plants. Lloyd's course covers only a half year's work. In content and point of view it does not differ radically from Part I of the course under discussion. It would hardly be fair to assume, though, that if Lloyd were to add another half year to the course, it would correspond closely with Part II of the course under discussion, for he is very strongly imbued with the idea of making the ecological phase strong in the high school. He says: "Morphology has no meaning apart from physiology, and a synthetic treatment of these leads to ecology."

It may appear that the writer has talked on both sides of the question, and, indeed, this is the case. With no experience in the high school, he is obliged to discuss the question from the theoretical point of view, and in so doing he considers it best to present the two most widely differing views of what a high school course in botany should be. One of these is fairly represented by the course under discussion.

To summarize the arguments that the advocates of the course might give:

1. The course includes those topics in the leading divisions of the subjects that are considered fundamental.

2. It therefore gives a comprehensive view of the subject from the ecological, physiological and morphological side.

3. It furnishes an excellent basis for advanced work in the subject.

4. Since it treats the fundamentals of the subject, it gives the knowledge most useful to one not pursuing the subject further.

5. The teacher is left free to pursue the method that brings the greatest cultural value.

To summarize the arguments of the nature study faction:

1. The material of a nature study course is not foreign to the child, but is found in his environment, and therefore sets the problems naturally.

2. The course, while it would not lead to a complete classification of the plant world, would give much insight into the character and interrelation of the plant groups.

3. The course would of necessity emphasize the adaptive and functional features of plants and hence lead to a comprehensive view from the physiological and ecological sides, which are the phases most readily grasped by the child mind.

4. The method of such a course is of necessity such as to beget a scientific habit of mind and natural and growing interest in plant life—qualifications which at once fit for college and for other phases of life.

William Crocker.

In harmony with the democratic spirit of our people the schools are organized to promote the welfare of the greatest number.

In the common school it is generally held that a study and a more or less perfect mastery of "keys" to learning is best for the future laborer in

any field whatsoever, but no such uniformity of opinion exists concerning the work of the high school. Some, whose opinion is certainly entitled to consideration, hold. in view of the fact that so few students comparatively go beyond the high school, the secondary school should offer courses that apply directly to the student's later career. On the other hand, many prominent educators hold that the best results are obtained by pursuing a course which is essentially cultural and preparatory to college.

The high school has catered to the first notion by an elective course of study, manual training, etc., and to the second by holding all students to a definite amount of work in some lines.

Agreeable to this second opinion, high schools and colleges have been seeking closer relationship; this movement has resulted in the formation of associations of colleges and secondary schools, of which "The Association of Colleges and Preparatory Schools of the Middle States and Maryland" is one of several examples.

This association, actuated by the desire to bridge over the chasm between colleges and secondary schools and possibly increase the number of students, instituted a "College Entrance Examination Board," whose duty it is to define the aim, outline the course to be followed by the secondary schools of the Association, and to plan an examination according to the foregoing outline, the results of which shall form the student's credentials when he enters college. The answers are graded on the scale of a hundred; the answer books are either returned to the applicant or sent to the college of his choice, which college fixes the grade that is acceptable to it for entrance. Examinations are held in every State and Territory in our land and every school of standing except Harvard has come to accept the Board's examinations as equal to their own, while ten have altogether done away with their entrance examinations.

The work of the Board during the three years of its existence has shown a marked and steady increase both in number of students examined (500 for 1901, 1,620 in 1903) and in number of colleges accepting its work (beginning in 1901 with members of "Association of Colleges and Preparatory Schools of Middle States and Maryland, and in 1903 including all the highest institutions in the United States except Harvard).

Botany was added to the list of subjects in 1902. Zoölogy was considered at the same time but not added, owing to the fact that not enough colleges designated it as a subject required for entrance.

The course, the outline of which you have before you, is open to two criticisms. In the first place, it seems to me that the various phases of Botany—anatomy, morphology, physiology, ecology and natural history of the plant groups—are too much isolated; that the first three, purely as such, have little use in the high school so long as but one year is devoted to the subject and we have to struggle with large classes. I have found that my classes are more interested and grasp the subject better if we take up the first three phases and part of the fourth as we study the plant

groups, *e. g.*, we study photosynthesis as early as it appears distinctly (in spirogyra by experiment) and note the modifications as we ascend the scale. I find that the study of plant societies and zonal distribution makes an excellent capstone for the year's course.

In the second place, I feel that the course covers too much ground. It seems to me that the material outlined is such as is covered in not less than the first one and one-half years in some reputable colleges. In view of the facts that botany is generally, or at least frequently, confined to the freshmen and sophomore classes in the high school, that eight hours a week marks the general limit of time given, and that the classes are large (each pupil therefore getting little attention), I believe that I have never seen the high school that could cover the work adequately. I would like to call your attention to some figures taken from the annual reports of the Secretary of the Board. I wish to compare the private school, which can give the maximum time and personal attention to the student, with the public school, in which, because of numbers, the individual student receives almost the minimum amount of time and attention, the first certainly being better able to cover the outlined course than the second:

		1902	1903	
Number of candidates	Private School.......	600	566	
	Public High School...	361	594	
Rating	90-100....	20%	...	Number of students making the preceding grades.
	75- 90....	...	12.5%	
	60- 75....	40%	50%	

As the number of high school students increases in proportion to the total number examined, the grading falls off, which points to the fact that the course is too heavy for the high schools to carry adequately. Of course, our figures extend over too short a period of time to be very trustworthy, but surely we may accept them as an indication of a tendency.

I have felt and now feel strongly the need of doing away with the cross purposes of high school and college. The institution of this Board is a step in the right direction, and although I believe that the course outlined in botany is too heavy to be well done, either by school or pupil, I wish to express myself as heartily in sympathy with the spirit of the movement.

T. J. HEADLEY.

Is the suggested coures of study the best for the high schools? Before we can discuss this subject intelligently we must inquire what the purpose of the high school is—whether it is a training school for the college or a people's school. High schools are now generally admitted to be training schools for citizenship rather than training schools for colleges. The "people's college" is a phrase we are now hearing applied to them. The reason for this, of course, lies in the fact that a very large majority of high school pupils never enter college. Still another thought is coming into favor—that the best training for citizenship is also a good training for the college.

It is not our purpose to discuss these questions, but we take it for granted that the work of the high school is to lead toward citizenship. With this in mind we may ask, What are the elements of a good course in botany which will give the desired results? My answer to this question will be best answered by asking and answering another question, viz., What shall be the aim of the course? Then, having done this, we may examine the course suggested by the committee of the colleges to see if it fulfills our requirements.

We think a good course in botany should, first of all, give training in scientific methods. The spirit and purpose of scientific inquiry should be implanted in the minds of the pupil by the work of the laboratory. Second, the course should give some conception of the laws of evolution working through plants. Third, the course should give information in regard to plants. Under this I would include these topics: How the plants live; how they relate themselves to their surroundings; how they are related to animals and to man. An acquaintance with common plants, both through laboratory studies and through field observations. Finally, the course as a whole should leave an abiding interest in plants in the pupil's mind.

Now, we may inquire whether the proposed course fulfills these conditions. To my mind it does admirably. There are few criticisms I can make.

If the course is to give an adequate idea of the dominance of evolution, the natural history of plants by groups should be given first, and not last, as recommended by the committees.

It has been said, "It is possible to erect a house and afterward put a foundation under it." But there is considerable loss of energy and orderly habit by such a plan and it certainly will not give such an impressive picture of the development of the plant kingdom.

There might be much greater stress laid upon the importance of plants to man without detriment to the course for the college. It always gives an added interest to anything when we learn how it enters into our own lives. Take the case of bacteria yeast, mildew, molds, rust, medicinal plants, and many others. A knowledge of these gives the pupil information that is really of much value and it gives also an increased respect for the study.

The physiology of plants is given much prominence in the course, due probably to the Chairman of the Committee on Botany, Prof. W. F. Ganong. A good deal of the work suggested in physiology can not be done by the pupils individually in the ordinary high school. The equipment required is expensive and there is not time to do that and all the other work of the course adequately. But it may be done by a semi-demonstration method, *i. e.*, the teacher sets up the experiments while the pupils note the data and results. This does not require so much time and apparatus and gives fairly satisfactory results. I should say also that there should be more emphasis put upon field work and acquaintance with plants in the field. This is why the old method of studying botany by identifying and labeling plants gained

such a hold upon the schools. The pupils had to get in the fields and see the plants, and many of them in this way came to love the plants. With the advent of the laboratories the importance of this field work and acquaintance with plants was unfortunately lost sight of for a time. The keynote of the work of botany through the spring quarter should be acquaintance with the plants in the field.

But the changes which I am suggesting can all be made by any teacher without detriment to the course as planned by the committee, and I may repeat what I said at the outset, that the course is admirably planned. In my estimation it is a good course for the high school and for the college, even if the minor changes I have suggested are not read into it.

W. W. WHITNEY.

NEW YORK STATE SCIENCE TEACHERS' ASSOCIATION EIGHTH ANNUAL MEETING.

(*General Sessions.*)

The midwinter gathering of educators at Syracuse in December was marked by two new features. For the first time the larger organizations, the Associated Academic Principals, the Council of Grammar School Principals and the Science Teachers' Association issued a joint program, a harbinger, perhaps, of future federation. The city of Syracuse, through the Chamber of Commerce and the University Club, gracefully extended its hospitality in a reception to the whole body of teachers. In point of ability to take care of visitors as well as of accessibility, Syracuse has demonstrated her superiority over other cities as a place for the winter meeting.

The Science Teachers' Association held its meetings in the beautiful new high school building on Warren street. The session opened at 2:30 p. m., Monday, December 28. In the absence of the retiring president, Prof. E. M. Pattee, of Syracuse University, introduced the president-elect, Prof. Irving P. Bishop, of Buffalo Normal School.

The first paper on the *Preparation of the Teacher for Nature-Study Teaching*, by Mrs. Anna B. Comstock, Cornell University, Ithaca, owing to the illness of the writer, was read by her assistant, Prof. William C. Thro. Following is an abstract:

"There is no stumbling block left in the nature-study path except the unprepared teacher. She is overworked anyway, and she is afraid to try any new thing that involves extra work and possible failure. With arithmetic, grammar or geography as criterions, she knows that she must master a thing in order to teach it, and she thinks she must master all physical and biologic sciences before trying to teach nature study. But a wide knowledge is not so essential as is the *right kind of spirit* in teaching very young children.

"If a teacher be interested in just one phase of nature she is likely to be a good nature teacher. . . . This interest is the key which unlocks nature's storehouse. . . . When a teacher is thinking of fitting herself for work in nature study we always encourage her to devote her few spare moments and small surplus energy to knowing some one subject more or less thoroughly. We encourage her to study

the flowers or the trees or the ferns or the birds or the butterflies in a masterful way. . . . First of all she should understand the principles which underlie her subject. * * * She should know them by experiment and observation, not only in the study of one species, but of several. . . . A fact in nature simply learned by rote means nothing. The facts about life have to be lived with in order to become significant. . . . Supposing our teacher wishes to study birds, how shall she go about it? First of all let her emblazon on her banner this motto, 'A bird in the bush is worth two bird books in the hand.' Let her begin by observing some one bird, the more common the better. . . She must come by thinking and observing into the comprehension of the principles which underlie bird life.

"Environment, food, modes of locomotion, migration, correlation of parts, when observed and understood, beget new interest and lead to a knowledge of species and classification. Aid in such pursuits may be found in courses of study in various schools and in good books. But the teacher must learn *to think and observe for herself.*

"One of the very hardest things to teach a beginner in nature study is the value of a perfectly simple observation. If the teacher is the right kind she will at once see the relation between her own observations and their importance in the schoolroom. She will see how she can use each new fact to give the child a greater interest in his work, and she will see at once how to use it in making the regular work of the school stronger."

The paper was well received and elicited a lively discussion, in which Prof. C. S. Sheldon, of Oswego, and others, took part.

The next paper, *The Nomenclature of the New York Geological Formations*, by John M. Clarke, State Paleontologist, Albany, explained the evolution of the New York system of classification and the reasons which have led to the rejection of old and the adoption of new names for certain stratigraphical divisions. The paper filled a "long felt want" in the minds of many teachers of geology to whom and to whose pupils frequent changes in nomenclature had appeared confusing and unnecessary. Following is an abstract:

"The 'New York Series of Geological Formations' is an historic monument erected by the labors of devoted workers, and fortified by the unequaled investigations continued through more than a half century of that most zealous and distinguished paleontologist, James Hall, a figure which towers in the history of American science. It is and has been for decades a standard of reference for all students of the older rocks throughout the world, and while other classifications proposed for these rocks, contemporaneously or subsequently, have fallen to the ground, it has withstood all the attacks of time." The first part of the paper is given to a review of the historic development of this nomenclature, including a thoroughgoing appreciation of Ebenezer Emmons and a vindication of his Taconic system. "Emmons was right. There is a great system of rocks and a great series of faunas in New York below the Potsdam sandstone. Such a series of strata was located by Sedgwick in Britain at even an earlier day than Emmons' and was called by him the Cambrian. Over this word and its bearings was waged still another controversy as warm as that in America. Subsequently the remarkable fauna of these primordial rocks in Bohemia was depicted by Barrande, but by the testimony of Barrande himself and in the honest knowledge of all geologists it was Emmons who first showed from its fossils, the only infallible guide, that the Taconic system was

earlier than the New York series and does not include strata of immense thickness and tremendous importance because of their carrying the first known organisms of the earth. To Emmons and New York, then, belongs the honor of this discovery.

"The prime motive and purpose in our revision of the New York nomenclature of the geologic formations have been those of loyalty to the landmarks which our father set up, to prune off, so far as circumstances would permit, an intrusive growth on the foundation stones. The classification by the original state geologists was prophetic and almost ideal; . . . its integrity was marred by later propositions from without and threatened by improper combinations. . . . We have endeavored to return to the simple usages of our distinguished founders."

The geology of New York has not remained stationary since 1843. The work of analysis of our formations and their faunas has gone on during sixty years without an interruption. Our knowledge has been thus enormously broadened and refined. "It should be a matter of pride to us all that New York is more accurately known paleontologically and geologically than any equal area on the western hemisphere. I will go even farther. Last month I received a letter from the most distinguished of French geologists, in which he exclaims: 'The work of the geologists of New York has made the paleozoic formations of America better known than those of the old world.'

"When the waters of the great Appalachian gulf of Devonic times were depositing the sediments of the Portage group, they laid down sands in eastern New York along an estuarine and brackish bottom; through central New York ran the littoral zone of prolific marine life, in western New York was deeper water, with black muds among the fine and coarse grained silts. In eastern New York the organisms of the rocks are estuarine and wholly unlike the profuse littoral fauna of central New York, and these again fundamentally and entirely different from the fauna in the Portage rocks of western New York. These three distinct faunas represent three distinct geographic or bionic provinces and yet they were absolutely contemporaneous and the strata containing them are essentially continuous across the State. There is but one method of procedure in setting forth this condition in stratigraphic nomenclature. Each province must have its own series of terms and multiplication of units is required for precision of expression. All are Portage; there is but one Portage time the world around, but all are not the same Portage—one may be Oneonta, one Ithaca, and one Naples. What is true in such a case is equally true in others." Hence the multiplication of terms demanded by the more refined study of the old stratigraphic divisions. Progress will demand an increase rather than a lessening of these terms. The aim is to keep out, as far as possible, extra-New York names. "On account of the peculiarly long east and west coast line in New York during the period of deposition of the old rocks our classification is gradually becoming a nomenclature according to meridians. That of eastern New York will not concur except along broader lines with that of western New York, and this divergence of expression in terms is bound to grow with the progress of knowledge. . . . No one will carry all these terms in mind. I have a list of them constantly on my desk. . . . They are not for practice in mnemonics, . . . but they are the tools, the scalpels and knives by whose help the earth's crust is laid open, the names of the successive pictures which portray the wondrous story beneath our feet."

Discussion of the preceding paper was opened by Prof. Albert Perry Brigham, Colgate University, Hamilton. "I approach the discussion of Dr. Clarke's paper from the point of view of a teacher, knowing that he, as an expert, may see good reasons for changes which must be adopted

slowly, if at all, by the teacher. Geology is growing, and perfect uniformity in the usage of terms can not be expected. The fittest terms will survive; the rest will become fossils in the older literature. The textbook should be conservative in making changes. . . . Without question local geographic names must replace mineralogical or accidental names for the smaller units of formation and the briefer time intervals. As a teacher I would prefer but three orders of time interval: era, period, and epoch. Subordinate place names might be multiplied as found useful. The teacher need not teach with equal insistence all the nomenclature of the book. The lesser divisions should be taught in detail only as a few of the type localities can be actually visited or are personally known to the class. In our State the places are few where the student can not see two, three or more type localities and thus learn the principles of all geologic succession."

Mr. P. F. Piper, of the Buffalo Central High School, then said: "That geology is a progressive science is proved by the fact that our present schemes of classification are not permanently established. Teachers will welcome any changes which may simplify the existing confusion due to the difficulty of correlating the terms proposed by the students of the subject. While welcoming these changes, conservatism would suggest that old classic terms should not be displaced if it be possible to retain them. The mere retention of a name for the sake of the name, or because of local pride, or in order to honor the memory of former masters of the science, is unreasonable and absurd if a more exact modern term would better meet the requirements of the science.

"In some cases it is not clear that a change is for the better. The term 'Niagara' is a case in which the substitution of 'Lockport' would not be an improvement. 'Trenton' is equally well known and only serious objection to its continuance by competent authorities should warrant its displacement. There would be little, if any, objection to the reintroduction of 'Taconic' or the other terms suggested in Dr. Clarke's admirable paper. The teacher can readily adopt the new system of classification for class use, for the pupils can learn one scheme as well as another. Unfortunately the text books can not be so easily changed, and the mature as well as the immature student is greatly confused by his inability to correlate the two systems. There should be some compromise and an agreement as to the major units and the regents should not hold pupils responsible for anything else except the local subdivisions in the immediate vicinity of the school. We wish to thank Dr. Clarke for the very valuable work he has been doing and we desire to coöperate with him in every way possible."

At 4 p. m. and at 9:30 a. m. the following morning the section meetings were held in the rooms assigned to them, reports of which are given at the end of this report on the general session. From 7:30 until 10:30 p. m. the High School was thronged with teachers, the guests of the Chamber of Commerce and the University Club. Everyone voted the reception a most delightful occasion.

The general session was resumed at 11 a. m. Tuesday. The papers elicited much discussion, owing to the fact that the regents expect soon to issue a new syllabus in physics and chemistry.

The first paper was on *Chemistry in the High School,* by E. G. Merritt, of Lafayette High School, Buffalo, N. Y., of which the following is an abstract:

"No science touches the pupil at more points or is more closely related with human needs and the ability to supply them than chemistry. To be liberally educated one should know something of chemistry as well as

ancient history and heathen mythology. . . . As presented to the beginner, chemistry should be related to familiar facts and human interests. Excursions to factories where chemical processes are employed, studies of the life and work of great chemists, will add interest to laboratory and textbook work and make the science more real.

"The particular order and method of a course in chemistry must be decided principally by the age and advancement of pupils, whether in the first year or the last year of the high school. In all cases, however, the laboratory exercises should be supplemented by the use of a good textbook. While the laboratory work is real, it is often incomplete without the information that is contained in books. There is nothing magical about a laboratory as a teacher of chemistry more than about a Latin room as a teacher of Latin. The supervision and questioning of the teacher is necessary to keep laboratory work up to the level of an intellectual exercise. The laboratory may be misused by teacher and pupil in such a way as to make the course no better than the old-fashioned courses in which the teacher provided all the experiments and illustrations. The pupil must think, study, calculate, write and report results in the laboratory, not simply weigh and measure, or, worse yet, employ his time in recuperating from the brain fag of other and more exacting studies. The quiz is valuable in testing the results of individual experiments and showing their relation to the teachings of the textbook, in deriving general principles from these experiments and applying them to the problems of daily life.

"The laboratory work should be intensive rather than extensive, thorough rather than superficial. Failure in this respect prevents the recognition of our work as a preparation for college. The real object of high school chemistry is to develop the love of truth, to make the mind logical and to cultivate habits of experimental observation. The information gained, whether useful or not, is of secondary importance."

Desirable Changes in the Regents' Courses in Chemistry and Physics was the subject of the next paper, by Mr. George M. Turner, Masten Park High School, Buffalo, an abstract of which follows:

As the courses of the Board of Regents of the State of New York are revised once in five years, and the next revision should occur in 1905, it is proper that the Science Teachers' Association of this State take up the discussion of the topic in time, that courses acceptable to the instructors of physics and chemistry in the secondary schools of the State may be clearly presented to the Board of Regents for their consideration.

The present courses in chemistry and physics have done much to encourage laboratory work in schools where five years ago little or no laboratory work was undertaken, and to create sentiment in favor of a full year's work where previously only one-half year courses were taught. But as our point from time to time changes, so our work for the next five years should be in advance of that of the past five years.

As the trend of modern teaching in chemistry and physics is apparently to accomplish two things, viz., the cultivation of the scientific attitude of mind in the pupil and incidentally the teaching of utilitarian facts, it is but proper that the new courses in these subjects appreciate the situation and conform to the same. To do this it is not sufficient to simply extend to each pupil handling the apparatus of an "approved" school for so many hours per year the credit of twenty points in an examination. This does not put the proper responsibility upon the pupil for his utilization of this equipment. The pupil should be so stimulated by the nature of the course and the character of the examination that he will approach the examination from a different standpoint. His work and his examination should test his attitude of mind toward the subject by requiring in the examina-

tion reasons why certain precautions are necessary to make his experiment a success; how to proceed in order to avoid certain errors of work or judgment; to give satisfactory reasons why the temperature and pressure conditions should be taken into account in working gases; why some experiments and certain parts of some experiments necessarily produce greater errors than do other experiments, or other parts of the same experiments; why utilizing a piece of apparatus in one way rather than in another way will yield better results. Such tests as these tend to stimulate the pupil to right lines of thinking, and it is proper that our regents' syllabus and examinations should encourage this class of work and the examination be based on the same.

But, in justice to efforts of the smaller schools, where insufficient equipment, lack of room and excess work of the teacher renders but little laboratory work possible, I would suggest that the Board of Regents outline two courses in physics and two courses in chemistry and issue two examinations in the same subject at each examination date. By this means the schools of the State might each work along lines suited to its possibilities and each take an examination in keeping with its work. Surely more work and work of a different character should be expected from the school possessing five talents than from the school possessing only two or one talent.

The work of the more extensive and intensive courses might properly be based on lines closely allied to those laid down in the last "Definition of Requirements" of the College Entrance Examination Board. These courses and examinations have a recognized merit and possess a standard value among the colleges. In those schools that fit for college such a course would not only be of advantage in properly preparing the boys and girls who must enter college, but give those not going to college an excellent course in mental training as well as an accumulation of the associated facts.

Such a syllabus would modify the present syllabus quite materially by cutting out certain work, putting in new work and doing away with the terms "Part I" and "Part II." These latter mentioned terms appear to have outlived their usefulness, since they divide the subject improperly and give the pupil a false impression of the proper classification of the subject as a whole.

As a course for those schools that feel unable to do the work of the more strenuous course I would suggest an outline having the same aim of training the pupil into right methods of thinking and work as the previously discussed course, but making the work less comprehensive with fewer required laboratory experiments. These experiments might naturally be based on as simple apparatus as possible where satisfactory results can be obtained. A course of this character would keep the same aims in view as the other course, but require only such work as the possibilities of the smaller school will allow.

It would thus seem that by making the courses in chemistry and physics for the next five years as far as possible with a view to cultivating the scientific attitude of mind, coupled with the necessary facts to make this training possible, and by fitting the examinations and work to the possibilities of the school and the requirements of the colleges, a standard would be set that would be beneficial in its effects.

In the *Discussion* which followed, Mr. O. C. Kenyon, High School, Syracuse, advocated (1) the introducing into the course in mathematics some of the simpler physical laws; (2) the arrangement, along parallel lines, of class and laboratory work, with special emphasis on the latter; (3) the lengthening of the physics course to two years.

Prof. C. N. Cobb, of the Regent's Office, invited those who had suggestions to send them to him in writing, so that they might be considered during the year.

The first hour of the afternoon session was devoted to the Report of the Committee on Stimulants and Narcotics. It was expected that the report would be complete, but owing to the illness of two members the committee was again obliged to ask for more time, which was granted. Prof. Burt G. Wilder read the portion of the report which was ready. Since the submission of the committee's preliminary report in 1901 much new material regarding alcohol has become available. The final report is based upon that of the Physiological Subcommittee of the Committee of Fifty published in July, 1903. It deals principally with the effect of alcohol upon the organs of the body in the light of recent investigation. It also discusses the antagonism now existing between those responsible for the present temperance law and the teachers who know its practical workings, and recommends a modification of the law which will do away with friction. Mrs. Cora D. Graham, president of the Woman's Christian Temperance Union of Onondaga County, opened the discussion with an able presentation of the subject as seen from the W. C. T. U. viewpoint. She suggested a conference of the teachers and the temperance workers with the object of eliminating points of difference and securing coöperation. Prof. Edward S. Babcock, of Alfred University, followed. He advocated the teaching of temperance on ethical grounds and deprecated the virulent attacks made upon the committee by temperance extremists. A general discussion followed, in which Professor Green, of Colgate, and Dr. Wilder had a short passage at arms and in which the latter stated his position on the total abstinence question with great clearness and force.

In accordance with Mrs. Graham's suggestion a resolution inviting the several State educational organizations, the W. C. T. U. and the New York State Central Committee to send delegates to confer with the Committee on Narcotics to the Science Teachers' Association, with the purpose of divining means "whereby the teaching of physiology, hygiene and, in relation to them, the nature and effect of alcohol and other narcotics, in our schools, may be rendered more efficient," was proposed jointly by Mrs. Graham and the chairman of the committee, and, on motion, was unanimously carried.

The last paper of the day called forth a short discussion, in which A. G. Clement, of the Regent's Office, Professors Stoller, Smallwood, Merrill and others, participated. Following is an abstract of the paper:

Zoölogy in the High School, by George H. Hudson, Normal School, Plattsburg, N. Y.

"Zoölogy is so valuable that it ought to be given to every student completing the high school course, but today it demands that maturity of mind found only in the last year of the course. Historically and logically, physics and chemistry precede it. They are its foundation stones. Digestion, osmosis, assimilation, combustion and other fundamental life processes can not be properly understood without them.

"This is a peculiar period of the world's history. The foundations of old faiths have been weakened and there seems to be no new bread for the hungry. Millions of people are led like sheep after new faiths with which to quiet an uneasy mind or new patent nostrums to heal some diseased portion of their physical being. It is a sorry commentary on our system of education if we call *these* people educated. The science student should begin to grasp the idea of the comparative value of evidence. He will come to see that faith is more valuable the more complete and trustworthy the evidence on which it is based, until faith becomes knowledge. Few realize how much of a scientist's work is done through faith. Some hold that our evidence for molecules and atoms is still incomplete. Even the indestructibility of matter and the conservation of energy are called in question. Perhaps we can not prove that the newly discovered laws of nature are unchanging, but we have faith that they are, and it is on a faith of this type that we build a steel bridge—and a sound civilization. Teach that God's greatest gift to man is the power to discover the constant in the seemingly inconstant—that there are things in this world on which we *can* depend—it may help us to make men and women on whom also we can depend. To my mind there are many scientific courses that fail to place this stamp on the material that comes from the mill, and I believe that for this age it is the stamp of greatest worth.

"The course should proceed from one celled forms of life to higher types. The dissection of the simpler invertebrates, in high school work, pays better than that of the higher animals. Students who have done good invertebrate dissection will be interested in the deeper knowledge to be gained from vertebrate dissection, which they should have in their college course. The salt water aquarium is easily kept for a few weeks and well repays time and expense.

"Let me urge the necessity of teaching zoölogy as a science, just as chemistry and physics should be taught. We must introduce our facts not alone for their interest or value *as facts,* but for the purpose of demonstrating zoölogical or biological laws. We must not be content with merely presenting the law, but must use it in helping us to grasp the greatest and grandest of all modern concepts, the concept of evolution. A course of this character will be worthy the name of science and will make for a type of civilization distinctly higher than any world has yet seen."

The first part of the Wednesday morning session was devoted to business. The committees on Second Year Physics Syllabus and on Laboratory Course in Physiology reported progress and were continued until next year. Dr. B. G. Wilder exhibited an improved apparatus for illustrating the action of the diaphragm which delighted the meeting and was warmly applauded. On motion of Professor Pattee a new section of mathematics, to be called Section E, was created, and will have a part in future meetings.

A REQUEST.

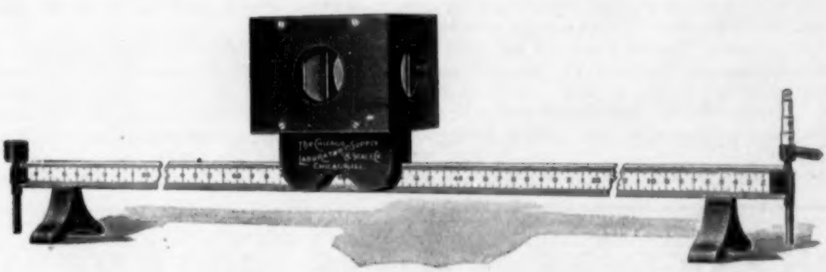
If any reader of School Science has had successful experience in organizing and conducting Natural Science Clubs for the purpose of out of school work in nature study in botany or zoölogy in secondary schools, he will confer a great favor by communicating with the undersigned.

A. J. Grout,
President New York Association of Biology Teachers,
Boys' High School, Brooklyn, N. Y.

SOUND AND LIGHT.

A convenient form of apparatus for demonstrating the phenomena of transverse waves is found in the electrically maintained tuning fork, a cut of which is shown below. This instrument is so adjusted that the amplitude of the vibrations may be increased or diminished at will. By attaching a cord to the prong by means of a screw, Melde's method of studying transverse waves may be readily demonstrated. This cord may be either passed over a pulley and weights attached, or held in the hand. The different wave lengths being obtained by varying the tension. This method will be found more satisfactory than that of the spiral spring or rubber cord, in that the same conditions may be maintained for any length of time.

This fork may also be used in the determination of the number of vibrations per unit of time, by means of tracings on smoked glass.



A simple and convenient form of the photometer is shown in the accompanying illustration. It consists of a meter stick mounted on two supports, having at one side a support for the source of light to be tested, and at the other a holder for a standard candle. Sliding upon this support is a box, painted dead black to prevent reflection, containing a screen, and two mirrors so arranged that the grease spot on both sides of the screen may be viewed at the same time. With this instrument accurate results can be obtained in a very short time, and its principle of construction and low price commend it for student use. One of the leading high schools of the country has ordered twelve of these instruments.

For prices of the above instruments, and anything else needed in a scientific laboratory, address C. H. Stoelting Company, 39 West Randolph street, Chicago, manufacturers and importers of scientific apparatus.

Mathematical Supplement

of

School Science

VOL. I. APRIL, 1903. No. 1.

EDITORIAL.

SALUTATORY In making its editorial bow to the secondary mathematical public the MATHEMATICAL SUPPLEMENT has no apologies to offer, and for the reasons which follow.

Every teacher of secondary mathematics as well as every student of the educational situation in the secondary school will agree at once as to the reality of the problems of the secondary mathematical teacher. For those actively engaged in teaching high school mathematics and science no argument is needed to prove the value of a closer coördination, of a completer unification of these two important lines of work than exists in most of the high school teaching of today. This need may therefore be taken for granted.

As to the ways and means of bringing about this desired coördination in practice, however, there is legitimate ground for difference of opinion. But there can be no great divergence of views among practical teachers as to whether the *what* and *how* of this coördination are to be answered at once or step by step, or as to whether the solution is to be theoretical or practical in its character. Few indeed who are familiar with our educational past would expect these questions to be answered otherwise than

gradually and on other than a practical basis. It is, moreover, safe to assume that not more than 10 per cent of our secondary teachers of mathematics and science are so firmly wedded to mathematical traditions and authority as to hesitate to reject the antiquated and to replace it by the timely so soon as the latter has been shown to be equally practicable and more highly educative. This is but another way of exercising the belief that the malady of pedagogical rococo is not sapping the vitality and virility of the scientific and mathematical member so seriously as it has sapped other members of the educational organism.

Inasmuch as no educational journal in America, or indeed in the English language, devotes itself exclusively to the province of bringing together in its pages in a unified and systematic way the results of actual attempts to improve secondary mathematical teaching through a closer correlation of both the matter and the method of secondary science and mathematics the MATHEMATICAL SUPPLEMENT is entering a substantially unworked and unoccupied field. It is not therefore pressing its claims for attention upon a reading public whose time for reading is already overfilled with professional claimants. The European teacher of secondary mathematics has at his disposition a number of journals devoted exclusively to the professional treatment of the practical problems with which he is dealing. Most prominent among these are *L'Enseignment Mathèmatiques* and *Hoffmann's Zeitschrift.* A European expert in secondary education once told the writer that these journals on secondary mathematics were not only generally used by secondary teachers but that they were more helpful to the active profession than was even the textbook literature in mathematics. American teachers generally can scarcely be said to take their journals so seriously as this. Perhaps this is because the average American contributor to professional journals does not take the real problems of teaching to heart so seriously as does his European counterpart. At all events, the customary American magazine treatment of educational questions too frequently—much more frequently than is the case in Europe—bears the ear-marks of haste, flippancy or desire for *éclat.*

Be this as it may, however, no open-minded, thoughtful and progressive American teacher would think of trying to get on in

his work without access to most of the leading educational journals which deal with his problems. One is not doing his best, for the reason that one cannot do his best, without knowing what his colleagues are doing and thinking concerning the burning questions of his time and profession. This is of course true of all professions, but it is true in a special sense of the teaching profession, if we may be permitted to speak of teaching as a profession. The SUPPLEMENT therefore makes so bold as to claim that it was conceived and has originated in a real and timely professional need which is both deeply and widely felt by the great body of secondary mathematical teachers. It will be the unceasing struggle of the editors of SCHOOL SCIENCE and of the MATHEMATICAL SUPPLEMENT to keep the pages of these papers—for they are in reality two papers in one—sacred to the needs of the secondary teachers of science and mathematics. To this end the SUPPLEMENT asks the hearty coöperation of every teacher of secondary mathematics whose pedagogical nerve is still sufficiently sensitive to enable him either to feel or to diagnose a real need. Contrary to the statements of many persons partially familiar with the situation as to secondary teaching, the editors believe that teachers of science and mathematics are more open-minded, alert and progressive than most other teachers, and that practical contributions bearing on the correlation of mathematics and science will be plentifully forthcoming. In the event of a surfeit our readers will get the very best.

Correlation of Mathematics and Science in the Secondary School

A criticism frequently heard and most generally recognized as forceful against current mathematical teaching in the secondary school comes from teachers of science. Their claim is that after students have gone through a year of geometry and a year of algebra and show considerable facility in handling the more or less cut-and-dried problems and equations of the mathematical texts, they fail utterly in the power to grasp the mathematical factors in the simplest quantitative problems of their science work and are hopelessly incapable of selecting

from the mass of undigested principles of mathematics they have been storing up, such particular principles as clarify, relate and organize the difficulties of any real problem called for in science. This is furthermore claimed to be true of mathematical pupils even after the scientific notions and nomenclature are fully comprehended by them. If this arraignment is just in the measure set forth in these statements, it will be difficult to convince students of the secondary school situation much longer that the results justify continuing algebra, geometry and physics in the front rank of claimants for time on the secondary school program, which the recent very careful analysis of Mr. Bonser of the State University has shown to be the case in the secondary schools of Illinois.

But making all due allowances for overdrawing the difficulties on the part of the science teacher, and for legitimate forgetting on the pupil's part, there still remains a core of truth in the arraignment that cannot be gainsaid. It is believed that the chief reason for the weaknesses of the present mathematical product lies in the too great isolation of the essentially cognate lines of study in the secondary school. The reason which appeals most strongly to the secondary pupil for the study of mathematics is that he must have it for his science. Mathematical teachers may dilate upon the beauty of logical perfection, the disciplinary value of mathematical thinking, the ethical value of the demonstrably true, the inestimable value of mathematics in later life; nevertheless the immediate demands of the right sort of physics, chemistry, etc., are to the student more real and more cogent, because more immediate, than any other reason, no matter how forcefully the former reasons may appeal to the teacher. At his stage of mental maturity the secondary pupil can at best look upon the former more abstract reasons as but secondary, easily replaceable by the values of other studies.

The most effective way then of bringing to bear upon the life of the pupil whatever value, or values, mathematical and science study have for him is through the organic unifying of the mathematical and allied scientific lines of work, now commonly carried on as isolated and not even as parallel lines of work through the high school course. This correlation must not be a mere inter-

locking, interrelating or dovetailing of courses; it must be an *organic unifying* of subjects which are bound together in their very nature and which have been separated only for artificial reasons. The final product must not be merely a *coherent course*, it must be an *organic unity*.

But all this is more easily said than done. It is not believed that such a unifying of courses at present quite distinct, without overstraining on the side of mathematics or of science, either of which would be alike fatal, without a painstaking, thorough-going and concerted effort of all teachers of all lines of study concerned. Many things must be tried with patient persistence and with steady continuity. Many attempts must fail. A few will succeed and pave the way for more. With perfect openmindedness the failures must be discarded and with steady courage the successful results must and will be adopted. So much then by way of clearing the ground for an attack upon the problem of finding a closer correlation of the mathematical and allied subjects of the secondary school.

As a definite step in the direction of attack, teachers of secondary science subjects everywhere are requested to send to us for publication as fast as we can find space for them, long or short articles on the question of what kind and amount of mathematical teaching would best fit students for their science work. A period of criticism and iconoclasm must be followed by a creative period, else the image breaking amounts to but little. Mr. Science Teacher, you say we secondary teachers of mathematics have not been giving you what you want. We believe you are correct, for even we ourselves are not satisfied with our product. Now be good enough to tell us precisely what you want and we agree to set about an earnest effort to "deliver the goods."

Without a knowledge of the point of view of the science teacher who believes in the use of mathematics in his science teaching, the teacher of mathematics must grope toward the goal of better teaching along a zigzag course, whereas with this knowledge he might fly directly thither. Let us have an armistice of criticism's destructive warfare and let us fill it with a concert of constructive effort directed toward the mutual betterment of both science and mathematics in the secondary school!

THE CONDITION OF SECONDARY MATHEMATICAL INSTRUCTION WITH SOME HINTS AS TO REMEDIES.

BY ELLERY W. DAVIS,

Professor of Mathematics, University of Nebraska.

It is a truism that in all education the attitude of the learner should as nearly as possible parallel that of the investigator. He should not so much be getting told what others have done as himself finding out what to do by doing, making use of what knowledge he has or thinks he has to lead to further ideas, further knowledge, and this both in the way of amplification and correction.

Yet what dc we find? Geometry, for example, is presented as a completed and perfected whole substantially as it came from Euclid. If problems are given, these are based not upon the student's ideas and conceptions, but upon the text and with the object mainly of getting the student to learn the text. To be sure, there are manuals written upon what may be called the leading-question method, but the questions always have in mind the old stereotyped form and could hardly be imagined to lead a student to build up a geometry of his own, selecting his own axioms, arrânging his own sequence of propositions, supplying his own analysis and criticisms, in brief, asking his own questions, and himself supplying the answers. Nor is instruction in algebra one whit better than in geometry. To the majority of students it must seem the art of juggling with symbols that have nothing but the vaguest of meanings. Almost on a par is it with the old method of learning Latin by memorizing the grammar. I can remember as a boy using an algebra by an author different from that used in the class and yet finding in it the same topics in the same order, and even the same numbers of examples to a topic!

No wonder the physicists have found fault with our teaching. With them the laboratory has been revolutionizing instruction. The investigating spirit has even crept into the high school. But when a next step would be taken, when the stu-

dent should apply the arithmetic, geometry, and algebra that he has been worrying over for years, he has no grasp and fails ignominiously. "We could do better with him by teaching him ourselves in our own laboratories," say the physicists. "Moreover, we could teach him physics at the same time."

Surely it behooves us mathematicians to be up and doing. Either we must turn to and help the physicists or they will supplant us. We must realize the connection that our science has with the other sciences and with life. We must keep ourselves and our science in touch with what is going on around us. Isolation means pedantry, and pedantry means death.

A step forward is it to correlate the arithmetic, algebra, and geometry, to introduce inventional geometry in the grades, to weed out useless technicalities, to simplify demonstrations. All this is good, much of this sort has been done. But it is only a beginning. Inventional geometry should be continued into and through the high school and we should have inventional algebra as well. Observing facts before reasoning upon them, guessing at other facts but always testing our guessing. Nor should this testing cease when the student reasons out his conclusions; for reasoning is but an outgrowth of guessing and, like it, is liable to error. Most useful is that test of conclusions, derived either by computation or by argument, that is furnished by various students working independently, using each his own method. We have in this, what the scientist values above all else, the concurrence of independent investigation, something that the routine following of a text totally fails to give.

Some may object that a text is necessary to give unity and system to the work. In such a way it can be made useful; but the guiding mind of the teacher, by the aid of conferences and the tabulation of results that the members of a class have themselves worked out and which they themselves help to tabulate, can secure a better system, because it is one growing out of the daily work. Moreover, by correlation with other subjects a system can be developed to embrace these also, the student thus learning not merely the detached unity of some fragment of mathematics, but getting a glimpse at least of the unity of all mathematics and all science.

But mathematics has its correlation with art as well. What would music be without rhythm; what would architecture be without symmetry and proportion? Many patterns and designs are plainly geometrical and others only less obviously so. The science of perspective, so useful to the artist, is but a simple development of geometry. Taught in connection with space geometry and accompanied by actual drawings from observation it would halve the difficulty and double the interest and value of the latter subject.

Then there are the numerous practical applications, such as that of plane geometry to surveying, to graphics, to the study of machines.[1] Such tables as the student needs he could mostly construct himself; his working formulas his own algebra would lead him to; and his own common sense should teach him, with perhaps some slight suggestion, that computation carried beyond the limit of accuracy which the data warrant is not only worthless but false and misleading. He will thus be prepared to appreciate justly the help afforded in computation by the slide rule, by graphical construction, or by logarithm tables of a moderate number of places.[2]

Perhaps the most important correlation of all is that of mathematics with clear English. It is on clear statements and their just and complete apprehension that the student's power of deduction depends. The mere careful writing out of a difficulty will half the time cause the difficulty to vanish. The student should be permitted to use symbols and technical words only so far as it is certain that he exactly grasps their meaning. To this end there must be constant exercise in translation into ordinary language from the technical and back, while even the everyday talk must be freed and purified from all that is ambiguous, meaningless, or misleading. The essence of mathematics is exact truthfulness.

[1]Castle, Workshop Mathematics. MacMillan.
Harrison and Boxandall, Practical Plane and Solid Geometry.

[2]Whapson and Gee, Mathematical and Physical Tables. Macmillan & Co.
Macfarlane, Elementary Mathematical Tables. Ginn & Co.
Von Velzer & Shutts, Place Tables Tracy, Gibbs & Co., Madison, Wis.
E. M. Longley, Computation. Longmans, Green & Co.

Besides the work and along with the work there may be mathematical play as well.[3] The old "puzzles" and "catch problems" had their place. No better way to teach the meaning of words is there than a problem such as to hopelessly trip the student who has failed to catch that meaning. Very instructive are formal demonstrations proving absurdities as that $3 = 6$ or that all triangles are isosceles. Difficult problems or "riders" that none have to work but which any one can get extra credit by working serve to create and maintain enthusiasm. Occasionally a mathematical tournament might be arranged, the winner being he who can devise the greatest number of problems that he can himself solve but which no other can.

All this will require thought and trouble on the part of the teacher, but the results will amply repay it. Nor is it at all certain that more time will be required for such complete teaching than for the half or quarter teaching now so prevalent. On the contrary, the experiment in the correlation of mathematics and physics under Miss Long in the Lincoln High School,[4] indicates a considerable saving of time. Nor need we be surprised at this; for a knowledge of sciences, as of things, is largely a knowledge of their relations to each other, their differences and their resemblances, their bearings upon each other. To teach them in segregation is to shut from the student a good part of this knowledge.

If these proposals be called revolutionary, we reply that even though there be crying need of revolution yet we advocate "revolution only by evolution." Out of the present must grow the better, and out of that, in succession, the better still and best. A good beginning can be made by the introduction of tests and quizzes which shall serve the double purpose of bringing to the student's consciousness his deficiencies and of setting his inventive and critical faculties to work. A recent series of observations on the teaching of arithmetic[5] in the grades indicates the value of this method. I have elsewhere[6] given some specimens of the

[3]Ball. Mathematical Recreations. MacMillan & Co.
Lucas. Arithmetique Amusant. Gauthier-Villars, Paris.

[4]Educational Review, December, 1902.
[5]Forum, Jan., Feb., 1903.
[6]Bulletin New York Math. Soc., Oct., 1893.

sort of questions in geometry that could come into such a test. The object is not to find out which students may go on (of course those as good as the average are to go on), nor to in some more or less mechanical way determine a grade, though it will well serve these purposes; but to discover the habits of thought of all the students. Thus is one prepared to lead them to better habits. There should be tests in which the student answers on the spur of the moment, and tests in which he is given all the time he wants, a year, if need be. There should be problems suited to all capacities and every encouragement should be offered to all to work at them. Not infrequently a dull student will succeed when all the rest fail, showing, perhaps, that both his more usual failures and his exceptional success are due to his having simply a different mode of thought from the majority. it may be a better habit.

I said this would do for a beginning. It will be a break with formalism. The bond once broken, few, I imagine, will desire to return to the service of Dame Formality.

THE OUTLOOK FOR ARITHMETIC.

BY DAVID EUGENE SMITH, PH. D.

Teachers' College, Columbia University.

The movement begun nearly twenty years ago in this country, looking to the elimination of obsolete matter from arithmetic, has been more or less successful. Like all such movements, reformatory in character, it has also been destructive. Destruction of a decayed structure not only requires less exertion than the construction of a solid edifice, but if one is to build on the same ground, it is almost a prerequisite. For some time, however, we have simply been contemplating the dust and rubbish pile, glad that the old building is gone, but uncertain as to what should go in its place.

There have not been wanting suggestions as to the new edifice. It has always been true, and it is true still, that the first thought at such a time is to move some house from up the street to the vacant spot. In years past they moved a so-called

theory of numbers, inherited from Boetius and Nicomachus, down into the grades; they put into elementary arithmetic (and we still keep there, in spite of our period of destruction) an unduly large array of symbols of operation, filched from the early algebras and now more nearly obsolete than we usually think; and so, in general, the world has pushed the higher subjects down to fill gaps. We have seen the same attempt since the recent edifice has been demolished. There has been an insistence that there is now room for algebra in the grades, and for geometry, although why square root should go out and algebraic fractions should go in we have not heard. Now all this moving of the higher subjects into the grades may be the best educational policy, but it has not been done with any evidence of careful consideration; rather has it seemed a part of the world's very old policy of somewhat thoughtlessly filling the gaps by moving higher subjects down. That this is so appears from the fact that in spite of the urgent demands for help, there has not yet appeared a well-defined system of geometry for the upper grades, that seems to be at all at home among its sister subjects.

Every period of this kind is, however, followed by a period of thoughtful construction, and upon this period there seem evidences that we are entering. What is the outlook for arithmetic? Does it promise a thoughtless moving down of higher subjects, or a thoughtful introduction of a new range of work?

Unfortunately a very common phenomenon is now being observed, the offering of a panacea in the form of "method." Now method is a most desirable thing in education, but the word has always been abused, and the old term "laboratory method" has probably been, and seems about to be, even more abused than most similar expressions. In the first place we shall find a host of people content merely with "method," nurses telling their patients how to eat without giving them food. In the second place we shall see all sorts of stupid, lifeless, slow, individual work palmed off in place of real, live, work-shop labor, energetic with a sparkling *esprit de corps*. Hence it behooves all who propose to do much talking about "laboratory methods" to preface their remarks by a denunciation of these dangerous tendencies, and to seek some happier term for the plan itself.

But something beyond method is needed, and that is substance. What is the substance; not merely the upholstery of the new edifice? The answer must be, *the quantitative side of the life which the child meets, directly or indirectly, in America today.* In the first two grades this means the counting of his games, a beginning in the problems of the house, and some measurements in his manual training. It does not mean problems on marbles unless he is using marbles in his play, nor problems on apples unless the season or locality suggests some reality to the question. Problems on four marbles or on four apples, as usually stated, are about as abstract as problems on four, *per se.* It means that a rug-weaving problem in grade two may be far more valuable than a page of ordinary text-book problems. In grade four, where interest begins to center about the problem of food supply, a rich field opens in the farming life of the country, the cost of production, and the transportation to market. The boy's interest in a locomotive, the experience of children in the cars or on a boat, all suggest an interesting range of problems that may well replace the dead topics of our books. In the later grades the great industries of the country, the cost of mining ore, of transporting it, and of extracting the metal, the cost of manufacturing, the cost of growing cotton, of transporting and of spinning and weaving it, the ever interesting topic of the growth of our own country in all of its lines of activity, and the graphic representation of statistics, these are the matters that are to replace the dead topics of partnership involving time, equation of payments, and compound proportion.

Of course this is not new in theory. But neither in practice is it new, for it has not begun in any serious way, nor can it begin at once. It must come only by the efforts of our best teachers to introduce these topics little by little, to encourage the collection and publication of connected problems, and to experiment as to the proper topics for the various grades. No laboratory method is going to succeed without something to work with, and this something must not, in the long run, be disconnected problems; it must be sorted into groups of related problems. Think of a laboratory in which a student goes to one bench and weighs a test tube, then goes to a telescope and looks

at the sun, then hurries to a microscope to see the corpuscles in a frog's foot, and then is called to see some iron filings acted on by electricity! And yet that is what the laboratory method will degenerate into in arithmetic, unless we can group our problems, relate the groups to the child's other interests, and relate these groups not to nature alone, nor to trade alone, nor to the country's growth alone, but to as many sides of human activity as come at any given period within the region of the child's interests.

I therefore appeal for these results of our present healthy discontent with arithmetic as it stands:

1. Replace the eliminated material by *groups* of related problems, correlated as far as reasonable with the other work, but not touching merely one side of the child's life. The field is *the quantitative side of every interest the child has,* and if only the nature side, or the commercial side, or the industrial side, or the statistical side be touched, then there is a failure to secure the best results.

2. Let no laboratory or any other method draw us away from this question of material content of arithmetic. On the other hand, let no question of content blind us to the value of any plan of presentation that makes the subject tingle with life. Good teachers have always presided over laboratories; poor teachers will always make sorry work of it; but to the extent that a class is a genuine workshop, to that extent the subject will be real and the interest will be secure.

3. Let no question of applied problems and no question of method close our eyes to another vital question. The workman soon comes to see the need of tools, of sharpening these tools, and of machinery that is kept well oiled and in repair. He may not at first like to sharpen the tool or oil the lathe, but the results soon show him that his work is the better for it, and with this comes a joy in this less interesting labor. As applied to arithmetic, the habit of fairly rapid oral and written work must be acquired, and we shall never free ourselves from the necessity for abstract drill, at least until computing machines are more common and more comprehensive. Hence the

teacher who, in our generation, fails to interest children in abstract work, fails to make them skllful in "mental arithmetic" and in written computations, and fails to offer enough daily drill to keep the mental number-machine well oiled, cannot hope for success.

Never in the history of education has there been a better outlook for some genuinely good work in arithmetic. Never has there been a better opportunity for removing the just reproach that the subject is not taught as well as it was in the old district school, that good, old laboratory with a less pretentious name. With good content, with good form, and with a clear vision of the goal, the teacher of the next decade should regenerate this too neglected member of the old trinity of R's.

THE MATHEMATICAL LABORATORY

BY C. E. COMSTOCK.

Instructor in Mathematics, Bradley Institute, Peoria Ill

There is a widespread feeling, daily growing more insistent, that present methods of teaching mathematics are inefficient. Especially do teachers of physics point out to us the fact that the pupils who come to them are unable to apply the algebraic and geometric knowledge which they are supposed to have, to the problems presented to them. Those teachers of mathematics, who see and think, are becoming awake to the situation and many of them are searching for a better way. It is full time for us as teachers of mathematics to re-examine our ideals and methods.

Abraham Lincoln was accustomed to say that he laid the foundation of all his legal success by the study of Euclid. In that study he developed power to carry on a rigorous and logical course of reasoning; but in after years he used that power in a field entirely foreign to mathematics, in a field, indeed, where the reasoning is of a sort very distinct from that of geometry. For him, geometry was but a mental trainer; he learned to think. Such power is not native; it can be trained and developed; a man must learn to reason before he can reason well just as he must learn to ride or shoot. Such power must always be one of the results sought by a good education. Undoubtedly mathematics

furnishes a most excellent means of training in a certain kind of reasoning. This is true partly because of the precise nature of the materials with which it deals and partly because of the fact that the results of its abstract reasoning can be so easily verified by ear and eye and hand. Of course, in the flights of pan-geometry and pure analysis such verification is not needed by the trained mathematician. But for the learner the flight on untried wing must have such aid, or disaster is certain. I said reasoning of a certain kind; very little of the reasoning of life is of that kind. We may be certain of a geometrical fact, but not so of an historical fact supported as it is by human evidence. As La Place has well said, most of our reasoning results in greater or less probability, not certainty.

Mr. Lincoln did not use his Euclid to win his cases. But now-a-days science has so taken hold of human life that many a man must use his mathematics in future work; to many a science, mathematics holds the key and to applied science or engineering it is essential. Man seeks to master the natural world about him, to discover its hidden forces and facts and to make them act in his service. The physicist studies the lightning flash; the astronomer, the movements of the planets; the chemist, the air; the geometer, areas and volumes; the engineer, the ways of doing things; each in turn reduces the relations he discovers to precise statement. From that moment the method of his study changes; it becomes mathematical. We may say then that mathematics is a method of science; while the statement in precise terms determines the materials upon which the method may work. The more advanced a science becomes the more are its relations expressed in precise terms and thus rendered susceptible to mathematical treatment. Geometry is so thoroughly mathematised that sometimes we forget and confuse geometry, the natural science, with geometry, the abstract science. Mathematics is then the ultimate method of science.

But the method is itself independent of the materials upon which it works. The story is told of a certain youth in an algebra class who was asked to compute the number of oranges in a pyramidal pile of given dimensions. When he had completed his work, the professor asked him, "Well, Smith, how many apples did you find?" The youth, with a look of disgust upon his face, swept the

figures from the board, at the same time saying, "Oh, I thought you said they were oranges."

It is from the interdependence of natural science that we arrive at the abstract science of mathematics. But it must ever be remembered in our elementary teaching that the beginnings are rooted in a material science.

But the study of science was called out by human need. Science for science' sake, is the motto over the cobweb door of Dr. Think who lives in the fifth story attic, back. Sometimes the human need brings the discovery of the scientific fact; sometimes the scientific discovery reveals a human need. Science furnishes the tools with which the engineer works and the most powerful of these tools is mathematics. Algebra is not a complicated game of hocus-pocus in which you march your men up hill and then down again to that good, old tune "The rule says so." No, it is a living method by which you find the distance of the approaching storm from the time between flash and thunder clap, or calculate the movement of a pendulum. Geometry is vastly more than a logical ring master; it enables one to build yonder bridge or judge a bargain in watermelons.

In an address before the British Association a number of years ago, Professor Sylvester said, "I should rejoice to see mathematics taught with that life and animation which the presence and example of her young and bouyant sister (*viz.*, natural and experimental science) could not fail to impart; short roads preferred to long ones; Euclid honorably shelved or buried 'deeper than did ever plummet sound' out of all schoolboys' reach; morphology introduced into the elements of algebra; projection, correlation, motion accepted as aids to geometry; the mind of the student quickened and elevated and his faith awakened by an early initiation into the ruling ideas of polarity, continuity, infinity and familiarization with the doctrine of the imaginary and the inconceivable. It is this living interest in the subject which is so wanting in our traditional and medieval modes of teaching." I believe that such enrichment and living interest can come best in what we may call the mathematical laboratory.

The mathematical laboratory is for the study of mathematics and as such must differ in some respects from the laboratories of

physics and chemistry. Its nature and methods must be worked out with that in view. Keeping this in mind let me lay down a few propositions which seem to me to be of importance in this connection. I may say that these propositions are not the results of theory alone; they are being tested in the institution in which I am teaching.

1. There should be an intimate connection between mathematics and the problems it solves. If possible, and in very many cases it is possible, the practical problem or difficulty should arise first; the need of solution becomes evident and the method of solution naturally follows. The thermometer may start the idea of negative number; the balance or motion may introduce the equation; problems of falling bodies naturally follow and the quadratic equation is introduced. An attempt at its solution will show a difficulty which must be overcome. The answer to the question may be long delayed but there is an object ahead and mathematics becomes a road to a goal. Problems on composition of motion lead to the necessity of introducing certain geometric and trigonometric ideas. The graph and the elements of analytic geometry cannot long be kept out. Hundreds of problems from the whole range of science and engineering come pouring in giving incentive to study and throwing a flood of light upon the whole "briar patch" of elementary mathematics. The divorce of elementary mathematics from the practical problems of laboratory and life is worse than nonsense; it is mathematical hari-kari. Many experiments which are today performed in the physics laboratory can be transferred to the mathematical laboratory with profit to both, relieving physics and enriching mathematics. Again the problem suggested may be purely a mathematical one; the same principles hold; the need, the solution; a resulting unity of question and answer. After the bisection of an angle the problem of trisection naturally follows. A pupil finds the distance a wheel goes and he asks how far does a point on the tire go? The problem may stare him in the face for years; he learns to wait; and patience is by no means a valueless lesson.

2. Such a program necessarily plays havoc with the ordinary divisions into which mathematics has been dissected. What's the odds? To me this is a distinct gain. I actually find that a five-

weeks' old student takes wonderfully well to a graph and can use the *pv*-diagram with some degree of understanding; sine and tangent and cosine have no terrors for him a little later, and he comes to understand *things*. The elements of the so-called branches of mathematics are simple and aid in the understanding of each other; they are inextricably commingled in the solutions of problems. The student studies mathematics, not algebra, geometry, etc.

3. Modern apparatus should be employed. Physical apparatus are a necessity; machines, balances, force tables, reflection boards, instruments for measuring, the plane-table, the equatorial, squares, parallel rules, protractors (this would make us part company with Euclid and Plato. No one outside of geometry classes ever draws a perpendicular after the method of the books.) The slide rule, mathematical tables, calipers, and instruments for drawing higher curves have as much right in the mathematical laboratory as has the multiplication table, the compass or the straight edge. Nor must we forget the use of geometrical models, sometimes ready made models, but more often and better, models built up on a modeling frame by the student in class. No properly equipped laboratory can be without its colored chalks, pencils and inks.

4. The validity of abstract reasoning should be tested by ear or by eye or by hand. Sometimes the tests should come before, to act as a suggestion; sometimes after, that it may serve to verify and make proof certain. Often a very much lauded geometric demonstration founded upon axioms, proved theorems, etc., proves nothing to the student for the very reason that he has not yet learned to see the force of such reasoning. Such verification is of greatest aid in many a theorem as: The bisectors of the interior and exterior angles of a triangle meet in four points by threes. In algebra such tests may be made by reduction of arithmetic and by physical experiments. Computations verified by actual measurement soon convince a boy that the results of abstract mathematical reasoning are sure and he is the more willing to trust himself to the guidance of mathematical methods. The untrained sees no certainty in such methods. The river men sat on the bank and laughed to see a man trying to hold a river back with a spy glass on three legs. But the completed dyke convinced them.

6. Practice must be given so that proficiency may be acquired. The physics laboratory does not aim to make an expert in finding specific gravities; an experiment is made once and left when understood. In this lies a difference between the two laboratories; the mathematical laboratory must produce experts. The ease with which the student progresses will depend in a large measure upon his expertness in the use of what he has gone over. In this the laboratory is more like the musical conservatory.

7. Individual work should be encouraged. The capable student need not mark time while he listens to the bungling efforts of the dull. The laboratory method must alter our ideas of the recitation and break, it may be, some cherished notions.

8. An opportunity should be given for observation and discovery. How dreary it is to be told everything and then merely perform exercises like those already given! The teacher of chemistry has learned this; why should the teacher of mathematics be so fearful?

9. Familiarity with phenomena themselves is of the greatest assistance to the comprehension of the theories explaining them. The student of physics works with electricity before he is told what it is or it is not; and when he comes to the theory he brings with him some very definite ideas of how electricity works. Ask an old time geometry class some practical questions about angles, planes and cones and you will receive some very surprising answers. I was once very much amused with a student who was vainly endeavoring to find the surface of a ball. He could not apply the method of the book—nobody could for that matter.

Clear and definite thinking is more dependent upon clear and definite seeing than we are sometimes willing to admit. I have on the table before me a piece of paper with but one side to it. How many who are not trained in higher mathematics and have not seen such a surface can get a clear idea of it? The field of a magnet is mapped out by the aid of the compass needle placed at various points. The story is vividly told. Just so the regions of convergency and divergency of a series mapped out with colored crayon fix ideas which the algebraic solution left hazy and undefined. The greater the field of clear vision the greater is the power to see clearly that which is just beyond.

Every method that will produce this clearness of vision, this definiteness of outline, this certainty in reasoning, this usefulness of effort, this vividness of aim should be welcomed by the teacher of mathematics. Away with traditions; growth is always a breaker of traditions. The day is coming, it has already dawned in some schools, when the desire expressed by Professor Sylvester will be fulfilled. I believe that it is in the mathematical laboratory that these changes will be worked out. Here and only here can we succeed in answering the challenge. I firmly believe that the same kind of a transformation will be effected in the teaching of mathematics that has been made in physics, chemistry, biology and psychology. It means labor; it means successes and failures; but it means in the end a vivifying, an enriching, a focusing of meaning, which is foreign to us today, but which is essential. This is no vain prophecy; already have I seen the results of a beginning, a beginning made in a small way but nevertheless a beginning, and I cannot close my eyes to what they indicate.

ALGEBRA EVOLVED FROM THE LEARNER'S EXPERIENCE.

ARTHUR C. LUNN.

Associate in Applied Mathematics, University of Chicago.

The following paragraphs are intended to suggest the need of systematic development in courses of algebra in the high school, of a number of hints, due to different teachers, which have grown out of their efforts to make the transition from arithmetic to algebra more natural, and to relieve the latter subject of that abstractness in fundamental points which is apt to make it seem so remote from the other interests of teacher and pupil.

Probably most students obtain at last a more or less definite idea that algebra owes its extended interest and usefulness, as compared with arithmetic, to the introduction of certain new ideas with respect to the nature and representation of numbers, and to the persistent following out of certain other notions which have lain dormant, as it were, in the arithmetic already familiar. Chief among the notions of the latter class are the formal laws of

operation and the idea of the equation, borrowed from arithmetic so far as concerns the unsigned numbers; of the former, the use of letters as symbols of numbers, and the notion of the signed number, with the corresponding extensions of meaning of many important concepts.

But it may be doubted whether this sort of understanding of algebra, of the value of its characteristic ideas and methods, presupposing as it does a certain maturity of mind, is not reached by the student too tardily to relieve the dissatisfaction sure to be caused by the seemingly arbitrary character of the new concepts, as often presented; whether the great power and real simplicity of the new realm of ideas are not largely offset by their apparent artificiality.

Every earnest teacher of algebra has probably felt this difficulty and worked out some way of meeting it. It is therefore believed that an outline of some of the suggestions available may prove useful, by contributing to the interchange of ideas concerning problems in the teaching of subjects having abstract phases, of which the problem in hand may be taken as a type.

It is doubtful whether conscious attention to any great extent need be directed to the formal laws relative to the signless numbers of arithmetic until quite late in the student's career. The facts expressed by the laws of commutation, association, and distribution, as to whole numbers, are chiefly *elements of freedom in the process of counting,* and as such are unconsciously familiar very early; no one needs to be told that the total number of a set of things can be found in many different ways, the choice being guided chiefly by convenience. The mode of expression in speaking of *the* product or *the* sum of a set of numbers implies indifference as to the order of factors or terms; while the parentheses, whose insertion and removal according to the associative law, later complicated with the law of signs, are often employed in a tiresome sort of way for algebraic gymnastics, really stand for halting-places in what is intended to be an economic method of counting by groups; or perhaps they are simply signs which direct the patient reckoner to pause for a long breath before he pursues his weary counting way.

What is really required seems to be simply a sufficient reali-

zation of the freedom expressed in these conventional forms; to be gained best perhaps by counting in different ways, singly or in groups, a set of objects whose number is actually required to be known for other purposes. Thus, in Fig. 1 the area of the rectangle in terms of a small square as unit may be obtained in the forms:

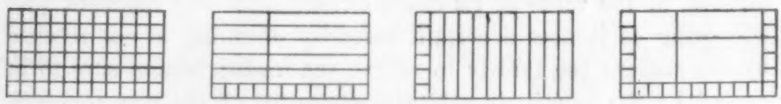


FIG. 1.

66 (by direct counting), 6×11, $6\times(4+7)$, $11\times(4+2)$, $(4+2)\times(4+7)$, and many others. Counting by groups serves to relieve the eye, while the geometric experiment indicated has another value in suggesting the general formula for the area of the rectangle.

The extension of this point of view to fractions is immediate, since the operations on fractions are reduced to those on whole numbers, in a way corresponding to a mode of counting with an aliquot part of the original single thing as unit. Illustrations are abundant in all kinds of denominate numbers. What is important, however, is that these concrete cases should be first considered, and the more abstract theory of fractional numbers left to be sifted out later on.

With the literal symbolism, the sign-bearing number, the widened meaning of the old words, the case is different; these are new ideas, not old ones more deeply realized. The most fruitful suggestion here seems to have been to have recourse to a method of development parallel with that of the original growth of algebra; to introduce each new concept *exactly when its need is felt* by the student as a result of varied experience in the employment of mathematical modes of thinking about objects and events in daily life which have a definite interest of their own, which are not merely hauled in by the ears to serve as unwilling witnesses to the divine right of certain vaguely seen and apparently usurping ideas to reign on the throne of abstract mathematics. From life to thought, from the varied details of every day's activity in seeing, hearing, feeling, to the powerful gen-

eralizations, abstract mental creations, of modern science, by a firmly trodden path, guided always by the desire to discover the most economical, most fruitful ways of thinking; such as this would seem to be the motto whose guidance has enabled many to release mathematical instruction at least in part from its isolation, and let it demonstrate the value of its functions in an organic educational system.

The conventions of algebra were made to use; they exist in their present form as the result of successive improvements of a mental tool forged at a time when perhaps no other function was dreamed of for mathematics; and even today this view may well be the dominant one in early work, giving place to others also, as fast as the student can be led to realize the significance, rational, esthetic, ethical, of an ideal in the sense of a perfected logical system of thought, gradually unfolded, and still in process of growth. Most students, however, never reach this latter view in the high school, if at all, and to them an emphasis solely on the logical aspects of mathematics acts more as a constraint than as an inspiration.

The use of letters seems to grow naturally out of the use of abbreviated forms of writing. In fact, it is useful to write equations originally in words, to shorten the notation gradually, and only after a fair acquaintance is gained with the nature of the equation to proceed to the conventional literal symbolism, which is thus to be justified solely by its greater convenience. Even then the best letters to choose are usually the initials of words in the concrete statement which is translated into equational form. Variety, flexibility in the choice of notation is a great help toward the proper subordination of all symbolism to understanding.

The general concept of the signed number is better left to ripen gradually out of an abundant experience in thinking and speaking about concrete magnitudes admitting of an opposition in direction or sense. The earliest statements about such magnitudes may well be written out in words, but as soon as a certain familiarity with literal arithmetic is available, one might have such equations as the following, referring to North and South journeys:

Na miles and then Nb miles amounts to $N(a+b)$ miles and corresponding ones in shorter form:

$$Na \text{ th } Sb, \text{ or, } Sb \text{ th } Na, \text{ gives } \begin{cases} N\ (a-b) \text{ if } a>b \\ S\ (b-a) \text{ if } a<b \end{cases}$$

Similar statements would apply to the rising and falling of the thermometer, N and S being replaced by R and F. For the forward or backward motion of a train, and of a brakeman upon the train, the signs of quality might be F and B, and the symbol of connection "&," since the two motions, of train and brakeman, might be simultaneous; thus:

$B12$ & $B4$ *m* $B16$ (*m* for makes)

The next step might be the use of the sign of equality to indicates "makes," "amounts to;" but this is an extension of meaning and should not be hastened. Following this is the recognition that the same two marks of quality might be used continually, to stand not only for "forward," "backward," but for any pair of contrasted directions, real or metaphorical. The two signs chosen should be suggestive; for instance, two opposite arrows, or the two dagger marks ↓ †. *It is better not to use at first the signs of "plus" and "minus" as symbols of quality;* their identification with the symbols of addition and subtraction is a fertile source of confusion, and is probably the chief trouble in the way of a common sense understanding of the addition and subtraction of directed magnitudes.

With respect to the operations on what will later be considered as signed numbers, the same spirit of treatment can be main-

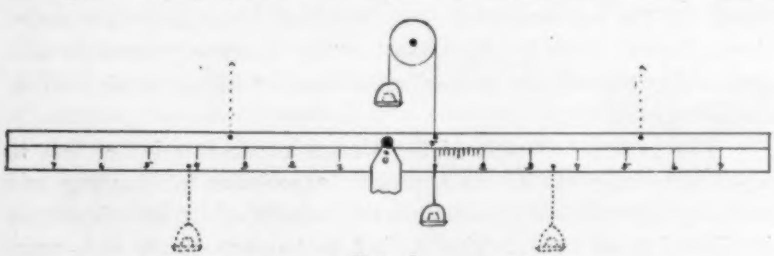


FIG 2.

tained; a single instance must suffice here. Fig. 2 gives the scheme of a lever pivoted at P, and a force is to be applied as a pull up or down on the right or left arm; the problem is to

determine the resultant force at the point F at the right, whose distance from the pivot is taken as the unit of length. Experiment gives: A force up of 3 pounds at a point 2 units to the right gives a resultant force up of 6 pounds:

$(Ua$ on $Rb)$ or $(Ub$ on $Ra)$ g Uab
$(Ua$ on $Lb)$ or $(Ub$ on $La)$ g Dab
$(Da$ on $Rb)$ or $(Db$ on $Ra)$ g Dab
$(Da$ on $Lb)$ or $(Db$ on $La)$ g Vab

which furnish one concrete suggestion of the need of the commutative and sign laws in the multiplication of signed numbers. *These laws are established by convention* precisely to make it possible to include all special cases in one general statement such as this: "The resulting moment is the product of force and lever arm." It is a matter of experiment to show that the same conventions are suggested wherever the product of two magnitudes occurs in the study of concrete phenomena.

But the abstract formulation of these conventions had better be postponed until most of the notions of elementary algebra are thoroughly familiar in this naïve, non-technical way.

PURE AND APPLIED MATHEMATICS.*

BY ELIAKIM HASTINGS MOORE.
Professor and Head of the Mathematical Department, University of Chicago.

In the ultimate analysis for any epoch, we have general logic, the mathematical sciences, that is, all special formally and abstractly deductive self-consistent sciences, and the natural sciences, which are inductive and informally deductive. While this classification may be satisfactory as an ideal one, it fails to recognize the fact that in mathematical research one by no means confines himself to processes which are mathematical according to

* This paper and two others which are to follow are extracts from Mr. Moore's presidential address delivered before the American Mathematical Society at its ninth annual meeting, December 29, 1902, *On the Foundations of Mathematics*. In the papers to appear in our next two issues Mr. Moore considers carefully and sympathetically the weaknesses of current mathematical teaching in elementary and secondary schools, blazes out lines along which effort at improvement should be directed in these schools as also in colleges, adduces reasons for the wide adoption of laboratory methods in mathematics classes, and calls upon friends of progress in mathematical methodology everywhere to join in a concerted movement for the energizing of mathematical teaching. The cause, as also its advocacy in these three papers, are so worthy and so timely, that our readers can profitably spend time for our next two issues, studying them. ED.

this definition; and if this is true with respect to the research of professional mathematicians, how much more is it true with respect to the study, which should throughout be conducted in the spirit of research, on the part of students of mathematics in elementary schools, colleges and universities. I refer to the articles ([9]) of Poincaré on the role of intuition and logic in mathematical research and education. It is apparent that this ideal classification can be made by the devotee of science only when he has reached a considerable degree of scientific maturity, that perhaps it would fail to appeal to non-mathematical experts, and that it does not accord with the definitions given by practical working mathematicians. Indeed the attitude of practical mathematicians toward this whole subject of abstract mathematics, and especially the symbolic form of abstract mathematics, is not unlike that of the practical physicist toward the whole subject of theoretic mathematics, and in turn not unlike that of the practical engineer toward the whole subject of theoretical physics and mathematics. Furthermore, everyone understands that many of the most important advances of pure mathematics have arisen in connection with investigations taking their origin in the domain of natural phenomena.

Practically then it would seem desirable for the interests of science in general that there should be a strong body of men thoroughly possessed of the scientific method in both its inductive and its deductive forms. We are confronted with the questions: What is science? What is the scientific method? What are the relations between the mathematical and the natural scientific processes of thought? As to these questions I refer to articles and addresses of Poincaré ([10]), Boltzmann ([11]), and Burkhardt ([12]),

(9) La logique et l'intuition dans la science mathématique et dans l'enseignement; L'Enseignement Mathématique, Vol. 1 (1899) pp. 157-162

Du role de l'intuition et de la logique en mathématiques; Compte Rendu du Deuxième Congrès International des Mathématiciens. (Paris [1900], 1902), pp. 115-130.

Sur les rapports de l'analyse pure et de la physique mathématique; Conference, Zurich, 1897; Acta Mathematica, Vol. 21, p. 238.

(10) In addition to those already cited:

On the Foundations of Geometry; The Monist, Vol. 9, October, 1898, pp. 1-43.

Sur les principes de la mécanique; Bibliothèque du Congrès International de Philosophie, Vol. 3, pp. 457-494.

(11) Ueber die Methoden der theoretischen Physik; Dyck's Katalog mathematischer und mathematisch-physikalischer Modelle, Apparate und Instrumente, pp. 89-98. (Munich, 1892.)

(12) Mathematisches und naturwissenschaftliches Denken; Jahresbericht der Deutschen Math.-Ver., Vol. 11 (1902), pp. 49-57.

and to Mach's *Science of Mechanics* and Pearson's *Grammar of Science.*

Without elaboration of metaphysical or psychological details, it is sufficient to refer to the thought that the individual as confronted with the world of phenomena in his effort to obtain control over this world is gradually forced to appreciate a knowledge of the usual co-existences and sequences of phenomena, and that science arises as the body of formulas serving to epitomize or summarize conveniently these usual co-existences and sequences. These formulas are of the nature of more or less exact descriptions of phenomena, and not of the nature of explanations. Of all the relations entering into the formulas of science, the fundamental mathematical notions of number and measure and form were among the earliest, and pure mathematics in its ordinary acceptation may be understood to be the systematic development of the properties of these notions, in accordance with conditions prescribed by physical phenomena. Arithmetic and geometry, closely united in mensuration and trigonometry, early reached a high degree of advancement. But after the development of the generalizing literal notations of algebra, and largely in response to the insistent demands of mechanics, astronomy, and physics, the seventeenth century, binding together arithmetic, and geometry infinitely more closely, created analytic geometry and the infinitesimal calculus, those mighty methods of research whose application to all branches of the theoretical and practical physical sciences so fundamentally characterizes the civilization of to-day. The eighteenth century was devoted to the development of the powers of these new instruments in all directions.

While this development continued during the nineteenth century, the dominant note of the nineteenth century was that of critical re-organization of the foundations of pure mathematics, so that, for instance, the majestic edifice of analysis was seen to rest upon the arithmetic of positive integers alone. This reorganization and the consequent course of development of pure mathematics were independent of the question of the application of mathematics to the sister sciences. There has thus arisen a chasm between pure mathematics and applied mathematics. There have not been lacking, however, influences making toward the bridging

of this chasm; one thinks especially of the whole influence of Klein in Germany and of the *Ecole Polytechnique* in France. As a basis of union of the pure mathematicians and the applied mathematicians Klein has throughout emphasized the importance of a clear understanding of the relations between those two parts of mathematics which are conveniently called "mathematics of precision" and "mathematics of approximation," and I refer especially to his latest work of this character, *Anwendung der Differential und Integral-Rechnung auf Geometrie: Eine Revision der Principien.* (Göttingen, Summer Semester, 1901; Teubner, 1902). This course of lectures is designed to present particular applications of the general notions of Klein, and furthermore it is in continuation of the discussion between Pringsheim and Klein and others, as to the desirable character of lectures on mathematics in the universities of Germany.

SOME RECENT DISCUSSION ON THE TEACHING OF MATHEMATICS.*

BY W. W. BEMAN.

Professor of Mathematics, University of Michigan

I was led to select this topic for this afternoon by the fact that at the Glasgow meeting, held last year, of the British Association for the Advancement of Science, the section on mathematics and physics and the newly organized section on education had a joint session at which the principal paper was read by Prof. John Perry of the Royal College of Science, London. This paper, as was intended, provoked a good deal of discussion, which was followed by the appointment of a representative committee with Professor Forsyth of the University of Cambridge as chairman, "to report upon improvements that might be effected in the teaching of mathematics, in the first instance in the teaching of elementary mathematics, and upon such means as they think

*Extracted from a paper read before the Mathematical Section of the Michigan School-masters' Club, at its 37th meeting, at Ann Arbor, March 25-29, 1902.

likely to effect such improvements." Professor Perry's paper was published in the *School World* for October, 1901, and the accompanying syllabus in the November number of the same journal. Later Professor Perry edited a book of one hundred pages, published by the Macmillan Company, containing his address, syllabus, reports of the discussion, together with written remarks from fourteen mathematicians and teachers of prominence who were not present at the meeting, and a reply by himself. I may add that the address was given in full in the *Educational Review* for February, 1902.

This discussion of the teaching of mathematics in England particularly with regard to the retention of Euclid as a text-book of geometry is by no means a new one. De Morgan, one of the best teachers of mathematics England ever had, in his article on "Mathematics" in the Penny Cyclopedia, and in substance in other places, says: "The work of Euclid is preferable in our opinion to any system which has been proposed to supply its place: simply because the dependence of conclusions upon premises is more distinct than in any other geometrical writing. The defects with which it abounds (and De Morgan was both logician and mathematician enough fully to appreciate them) are trifles which can be remedied as they are met with; and though there are seldom three propositions together, one or other of which will not call for some remark from the teacher, yet such is Euclid that these very faults, properly noted, are of more value than the greater elegance and more artificial process of less formally vigorous writers." On the contrary, Sylvester, whose enthusiasm bred contagion wherever he taught, whether in England or America, in words often quoted from his presidential address to the mathematical and physical section of the British Association in 1869, says: "I should rejoice to see mathematics taught with that life and animation which the presence and example of her young and buoyant sister (natural and experimental science) could not fail to impart, short roads preferred to long ones, Euclid honorably shelved or buried 'deeper than ever plummet sounded,' out of the school-boy's reach, morphology introduced into the elements of algebra—projection, correlation and motion accepted as aids to

geometry—the mind of the student quickened and elevated and his faith awakened by early initiation into the ruling ideas of polarity, continuity, infinity, and familiarization with the doctrine of the imaginary and the inconceivable." He confesses: "The early study of Euclid made me a hater of geometry . . . and yet in spite of this repugnance, which had become a second nature in me, whenever I went far enough into any mathematical question I found I touched at last a geometrical bottom." Again, Todhunter, the teacher, examiner and English mathematical textbook writer *par excellence,* in a 56-page essay on Elementary Geometry, wields the cudgels most vigorously in favor of a strict adherence to Euclid. He suggests that the grounds of Sylvester's dislike may have been "only the repugnance which might naturally be felt by a creative mind conscious of the power to advance without any superfluous aid," and says: "Tradition seems to record such characteristics of Newton and Pascal. It would be unwise, however, to suppose that such exceptional cases are likely to be common."

The opposition to Euclid in England seems first to have taken definite form in the organization in the year 1871 of the Association for the Improvement of Geometrical Teaching, whose object, as stated in the Code of Rules, "shall be to effect improvements in the teaching of elementary mathematics and mathematical physics, and especially of geometry." This association has published a syllabus of plain geometry, a syllabus of modern geometry, and a textbook entitled Elements of Plane Geometry; but by reason of the undisguised hostility of various boards of examiners, it has accomplished comparatively little. The present revival of the discussion is due to the untiring activity and zeal of Professor John Perry, whose charming naiveté is not concealed even in his textbook on the calculus. For twenty-odd years, in season and out of season, with reason, and sometimes, I fear, without reason, he has preached on "England's Neglect of Science," "The Defects in the Teaching of Mathematics," and similar subjects, till finally he secured a hearing before the two sections of the British Association in September, 1901. As early as July he notified those who were expected to take part in the discussion, of the

date assigned for the reading of his paper, September 16, and had copies of his paper printed for distribution before the meeting; but a change to a date three days earlier was found imperative, so that some of the expected speakers did not reach Glasgow in time, while those who were present did not have the wished-for opportunity carefully to examine the paper before the discussion. Still the written remarks from various contributors now published with the address, render this change of slight consequence.

Professor Perry was very much in earnest. He begged his auditors to give him the benefit of their severest criticism and advice, and said: "Anybody who thinks I am making a mistake, or who sees how my method may be improved, and who holds his tongue, is doing a real harm to the country." As would be expected from a teacher in a technical college, Professor Perry is a staunch believer in the utility of mathematics. The obvious forms of usefulness in the study of mathematics he enumerates as follows:

"1. In producing the higher emotions and giving mental pleasure. Hitherto neglected in teaching almost all boys.

2. (*a*) In brain development. (*b*) In producing logical ways of thinking. Hitherto neglected in teaching most boys.

3. In the aid given by mathematical weapons in the study of physical science. Hitherto neglected in teaching almost all boys.

4. In passing examinations. The only form that has not been neglected. The only form really recognized by teachers.

5. In giving men mental tools as easy to use as their legs or arms, enabling them to go on with their education (development of their souls and brains) throughout their lives, utilizing for this purpose all their experience. This is exactly analogous with the power to educate one's self through the fondness for reading.

6. Perhaps included in (5): in teaching a man the importance of thinking things out for himself and so delivering him from the present dreadful yoke of authority, and convincing him that whether he obeys or commands others, he is one of the highest of beings. This is usually left to other than mathematical studies.

(To be continued)

THE HELIOS.*

BY HIRAM B. LOOMIS, PH. D.
Medill High School, Chicago.

The course in physiography given in the first year of our high school curriculum offers a fine opportunity for the practical working out of the correlation of mathematics and science. This is certainly the place for us high school teachers to begin the correlation; besides there is the great advantage that laboratory work is being demanded in physiography and there is as yet no stock set of experiments. Merely as a suggestion of the possibilities in this direction I wish to call attention to a little

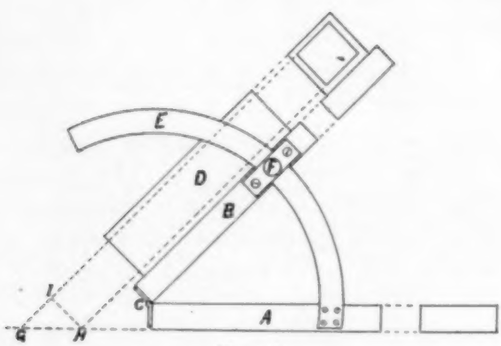


FIG. 1.

instrument now used by Mr. Joseph F. Morse, the teacher of physiography in the Medill high school.

The apparatus (Fig. 1) consists of two boards *A* and *B*, hinged together at *C*. Fastened to the top board *B* is a hollow tube *D*, of square cross-section. The tube may be set at any angle with the horizon and held in place by the screw *F*, while the angle may be read on the scale *E*. If it is desired a measurement in azimuth may also be obtained by pivoting the board *A* on another still below it and marking off a scale of degrees.

* An extract from a discussion on "The Correlation of the Teaching of Physics and Mathematics" before the Mathematical Section of the Chicago and Cook County High School Teachers' Association.

I shall call attention to but one use of this instrument, the helios, its use in comparing the intensity of the sun's heat for different altitudes. The instrument is so adjusted that the sun's rays fall directly through the tube D. Let us say that the tube is 2 in. square, making its area of cross-section (inside measurement) 4 sq. in. Then 4 sq. in. of the sun's direct light and heat pass through the tube; but, when this light and heat fall on a horizontal plane, they are spread over a rectangular space whose width is 2 in. and whose length is GH. If the area of this rectangle is 8 sq. in., then it is evident that 4 sq. in. of the direct light and heat of the sun are spread over 8 sq. in. and the intensity must of necessity be just half as great as when the rays are vertical. By taking measurements at different times of day, results may be obtained for different altitudes.

But it will not be long before the pupils will realize that a comparison of the lines IH and GH is just as good as a comparison of the areas given above. It will then be an easy step to pass from actual measurements with the helios to determinations by diagram, first drawing both the cross-section of the helios and the triangle GIH for a given angle, and then simply constructing the triangle GIH. A protractor will be needed to lay off the angle; a set square, to make the right angle; and a scale to measure the lines IH and GH.

After the pupil has become thoroughly familiar with the helios and with the construction of helios diagrams, some such problem as the following will pay from the standpoint of physiography as well as from that of mathematics. Give the time of sunrise and sunset for the longest and shortest days in the year in Chicago; give the altitude of the sun at hour intervals during those days, require the pupil to compare the amounts of heat received from the sun during those days. Such a problem will take time, will require study and thorough discussion; but it is well worth the time, for the boy who has intelligently worked such a problem knows something about climate, as well as some mathematics.

In the line of mathematics at least two things have been accomplished: First, though the words, sine and cosine, have

not been used, the things, sine and cosine, have been constantly employed; and the whole subject of the resolution of vectors, such as velocities, accelerations and forces, has been illustrated. Second, there has been training in passing from an experiment actually performed with material apparatus to one performed in the mind. And, in my opinion, this power to get a general mental experiment out of a few concrete experiments with apparatus is worth more than all knowledge of climate and all familiarity with sines and cosines. It is the very essence of all applied mathematics.

The above is simply one experiment with a single piece of apparatus and is given simply to suggest the possibilities of first year science in high schools. I believe that the principle of correlation which underlies this illustration furnishes us the key to a course of teaching which will not only give the maximum knowledge of science and mathematics but will at the same time furnish the best possible mental training.

Round Table.

Secondary mathematical teachers are invited to avail themselves of our Round Table page or pages, as circumstances may require, to communicate with their colleagues in the form of brief reports of experiments on correlated mathematical teaching, or of mathematical meetings, in the way of notes of suggestion, or ciritcism or commendation touching laboratory methods as applied to mathematical teaching, or of inquiries as to what others have found to be good methods for treating special mathemathical topics. Anything mathematical which is brief, pointed and purposeful will be welcomed. A teacher may here be of great service to his fellows without taxing his time with the preparation of an extended contribution.

The remark is often heard from teachers of mathematics that it is no part of the duty of a mathematical teacher to trouble himself with looking after the pupil's ability to use his mathematical knowledge. "Give the pupil skill in the manipulation of his mathematical symbols," say these persons, "and leave the science teachers to look after the applications to their own sciences." The writer believes this principle of action is responsible for most of the weaknesses of current mathematical teaching. Dr. Osgood, of Harvard, one of the foremost mathematical teachers and investigators of our time, is credited with saying that a student's ability to prove a proposition is no assurance that he knows it. The test as to

whether he knows it is whether he can use it. This principle, persistently practiced by the secondary mathematical teacher, will be productive of the most wholesome consequences. It will go far toward breaking down that isolation of mathematics so prevalent with us, which both cripples the student and devitalizes the subject.

The undersigned would be glad to learn whether there is any good reason to believe that boys and girls of high school age really do see any great force in deductive methods of proof. Does the idea of reasoning from certain fundamental axioms to their logical consequences make any appeal to such students? If so, how does it happen that the high school student is so ready to discard the most rigorously drawn deductive conclusion if it seems to contradict obvious relations in an erroneously constructed figure? Does not the pupil in the majority of cases accept or reject the conclusion as credible on grounds of intuitive plausibility, or non-plausibility, in most cases? When you get to the bottom of his thought process, does he not assent to the conclusion "because it looks all right?" If so, how much of our insistence on logical rigor in deductive geometry is of educational value?

Notwithstanding all the harsh words we hear about the use of the mathematical puzzle, there are times when a point can be driven home to a class through a puzzle which illustrates the point. For example, the writer has often used the following "catch" to bring clearly before the pupil's mind the importance of retaining the negative sign before an even root (in this case the square root) in algebra. Write

$$\tfrac{25}{4} = \tfrac{25}{4}.$$

Then subtract 6 from both sides,

$$\tfrac{25}{4} - 6 = \tfrac{25}{4} - 6.$$

But -6 may be replaced by either $4 - 10$ or $9 - 15$, thus:

$$\tfrac{25}{4} - 10 + 4 = \tfrac{25}{4} - 15 + 9.$$

This may be written:

$$(\tfrac{5}{2} - 2)^2 = (\tfrac{5}{2} - 3)^2.$$

Extracting the square root of both members:

$$\tfrac{5}{2} - 2 = \tfrac{5}{2} - 3.$$

Subtracting $\tfrac{5}{2}$ from both sides and changing signs,

$$2 = 3.$$

Query: Where was the error made?

Of course, by using the $\pm$ sign and remembering that we can infer from the equality of two squares only one or the other of two things, in this case that the positive root on one side equals the negative root on the other, the difficulty disappears. Thus

$$\tfrac{5}{2} - 2 = -\tfrac{5}{2} + 3$$
$$\therefore \quad \tfrac{1}{2} = \tfrac{1}{2}.$$

Every teacher has found devices of this sort which serve to emphasize some particular point which is the source of confusion, or difficulty, to the student. May we not ask that such devices be made more generally available to the teaching public through our "Round Table?"

Perhaps no living man has shown greater persistence in the attempt to bring clearly before the public attention the shortcomings of the mathematical teaching of his day or greater energy in the struggle against all odds to improve such teaching than has Professor John Perry, of England. For well nigh a quarter of a century, in season and out of season, he has been fighting the organized worshippers of Euclid with some success and more failure until even the Oxford and Cambridge dons are beginning to lay an ear to the ground. The wave of interest in improvement of mathematical instruction recently aroused in England mainly through his tireless efforts has even sent a very perceptible wave to our shores. All teachers who believe in the need and possibility of improvement in mathematical instruction will be pleased to learn that Prof. Perry has accepted the invitation of the University of Chicago to deliver a course of public lectures at the University during the coming summer. A real, live apostle of mathematical reform! and from the land of Euclidean idolatry!! Think of it! Sheer curiosity should make a thousand miles seem short to witness so unusual an occurrence!

M.

METHODS IN MATHEMATICS.

The high perfection which mathematics as a science has reached impresses the student, who undertakes to learn it in its perfected form, with the feeling that it is exceedingly stereotyped and severe. Teachers, as a matter of fact, are often led to adopt equally severe and formal methods, from the feeling that methods used elsewhere in science teaching are not permissible here. A lecture on a mathematical subject illustrated by stereopticon—how absurd! But is it so absurd after all? Would not the following subjects, when properly illustrated, be interesting and stimulating to pupils in our secondary schools?—say the

development of the modern symbolism of the equation; its gradual growth from the full sentence, then the syncopated forms, and, finally, the various symbols which have had their vogue up to the present? An industrious teacher may perhaps be able to write a given equation in all the modes; I hope to try it some day, but others, no doubt, can do better. The numerous ingenious devices to represent powers will arouse interest—such as *a-a-a for* a^3, or *a cubus,* or, if an unknown quantity, (3) —a circle with a figure 3 within.

In numerical work, we might show the Roman calculator with his dust-strewn tablet and pebbles or "calculi," the Chinaman with his swan-pan, the necessity of the cipher and its late invention, devices for computation, Napier's Bones, tables of multiplication which preceded logarithms, even logarithms and modern adding machines. The geometry teacher might show the fifty or more possibilities of a simple geometrical construction, thus emphasizing, without the loss of much time, the wide application of a single method.

We are constantly reminded of our great material progress in the last century. Do we not appreciate our telegraphs, railways, trolley cars, etc., the more because we are aware that they were not always with us, and we can, in a way, picture conditions as they might be without them? Might we not, in like manner, appreciate all the more our numerical system, our symbols and whole mathematical machinery, however ancient, if we are reminded that they were not always thus? Surely, it seems worth trying.

Jefferson High School, Chicago. C. A. PETTERSON.

Book Reviews.

Higher Arithmetic. By WEBSTER WOODRUFF BEMAN and DAVID EUGENE SMITH. 13x19 cms., xvii+193 pages. Ginn & Co., Boston, 1902. 80 cents.

This extremely valuable little book is intended for use in a completing course in arithmetic. It is singularly free from traditional arithmetical puzzles, is packed with problems drawn from the real conditions of daily life and with practical suggestions as to methods of checking computations, of attacking problems and of correlating arithmetic and algebra into an organic unity. The chapters on logarithms, graphic arithmetic, longitude and time, and on business arithmetic deserve special commendation, both as to matter and method of treatment. Without the customary pedagogical preachment so characteristic of recent arithmetical texts, whose pedagogical value is in sore need of dogmatic props, this book is the most truly pedagogical, because the most common sense arithmetic the reviewer has seen. The writer does not

know the extent to which this book is used, but he feels safe in the assertion that it deserves a much wider use than it has hitherto been given. It is clear, concise, stimulating and informing, while it sacrifices neither mathematical rigor nor naturalness in securing these desiderata. It is unhesitatingly commended to any in need of a text for final review and completing courses in arithmetic.

M.

Practical Lessons in Algebra, Complete. By JOSIAH H. GILBERT and ELLEN SULLIVAN, Teachers of Mathematics in the High School, Albany, N. Y. 13x19 cm., vi + 316 pages. Richardson, Smith & Co. 1902.

This book is an attempt to present elementary algebra by the inductive method. The work is arbitrarily divided into 197 lessons, with 104 miscellaneous examples at the end. Preceding each lesson are instructions for the teacher and pupil. In general, the teacher is advised to show that certain principles and theorems are true by means of various special cases; then the pupil is asked to solve the problems in accordance with these rules. But few of the rules are formally stated. A curious inconsistency is the statement and deductive proof of all the ordinary theorems of proportion. The book contains much method and little algebra outside the long lists of problems set for the pupil. It is, in reality, a manual for the class room.

WILLIAM O. BEAL.

Reports of Meetings.

REPORT OF MEETING OF TEACHERS OF MATHEMATICS OF THE COOK COUNTY AND CHICAGO HIGH SCHOOL ASSOCIATION.

The meeting was held in the rooms of the Board of Education, Tribune building, Chicago, on January 3, 1903. The editors have been unable to obtain a complete report of the meeting, but the revised remarks of all who have thus far sent them in for insertion are given below.

The meeting was called to order by the president, Mr. C. A. Petterson.

Messrs. Furlong and Loomis read papers. Mr. Loomis' paper is found elsewhere in the *Supplement*.

MR. N. J. LENNES, John Marshall High School, said: While most of our first year algebra, for instance, may at first glance appear to be

abstract mathematics, it is really nothing of the sort. Most of the formal operations are nothing more than juggling with letters. Much of the teaching, which is effective enough in getting pupils to pass the tests sent out from the Board Rooms, is the exact equivalent of having young people commit to memory quotations in an unknown language.

What is desired is a subject matter of some kind. A set of problems is contemplated arising out of direct contact with the material world, physics, physiography—science in general. In the solution of these problems, we shall find a real justification for building the various parts of our algebra. Then the algebra would become a language in which to express our knowledge of the world, and an instrument with which to investigate that world, while at present it is, to a greater extent than we ordinarily think it, a mere collection of meaningless expressions that are to be memorized.

Mr. C. R. Mann's remarks were: We are all familiar with the mental attitude which was characteristic of the Greeks, namely, that they were different from the rest of mankind; for were not they *Greeks*, and the rest of the world barbaroi? Was not the same spirit maintained by the Romans as manifested in the high esteem in which Roman citizenship was held? To become a *Civis Romanus* was to be elevated from the realms of the ordinary to a position which differentiated one from the rest of the world. Thus one of the fundamental conceptions of the Greek and Roman civilization was the idea that humanity was divided into numerous small races or groups of men having little in common—whose chief occupation was to war with each other.

In sharp contrast to this is the central conception of modern civilization. For are not all nations externally bound together by the steamship and railroads and the telegraph, and internally by the feeling of the unity—the solidarity of mankind?

If we apply this contrast to education, must we not confess that we educators are still holding fast to the classical ideal? Do we not regard the separate subjects of study as individual separate things and too little as parts of one unified development?

But if we wish to attain greater unity, how can it be done? It seems to me to be possible by paying regard to the bonds of unity which exist between the various subjects. These bonds are very numerous, but there are two which seem to be of especial importance. First, the unity of the sciences lies in the scientific method. Hence, if we science teachers would link the various branches of science into a unified whole, we must so present our subjects that the student is impressed with our method of thought rather than with our facts and fallacies.

Second, the unity of all thought lies in the principle of evolution by which the different phases of thought have developed. This principle can be studied only in connection with a true history of thought—not a

history of discoveries and inventions, but one in which the life of the subject is manifested by a discussion of the ideas which are fundamental in the subject at different epochs.

Hence, the two factors in our instruction which seem to need greater emphasis if we desire to bring the educational ideals in science up to date are: (1) The scientific method of thought; and (2) the true history of scientific thought.

Miss Mabel Sykes, South Chicago High School, remarked: I am sorry Mr. Rice, of Armour Institute, and Mr. Woodworth, of Lewis Institute, are not here today. They have given me a clearer idea of what is expected in this movement than anything else has. As I understand it, Mr. Rice's work is in connection with the night school. He is giving short, parallel courses in mathematics and physics, so planned that the one correlates with the other. Mr. Woodworth's work is regular day school work. First year pupils are given five recitations a week, four in algebra and one in physics; second year pupils, four in geometry and one in physics. In each case the mathematics classes are mathematics classes as ordinarily understood, and the physics classes are physics classes, only the work is so planned that one helps the other. I have regarded this movement with considerable suspicion, for I fear the effects of dissipation of energy, which must come if one tries to do two things at once. While I have great faith in cumulative results, I also believe in concentration of effort on one thing. Perhaps one may infer from the work of these gentlemen that we are not expected to teach two subjects in the same recitation.

We are told that the work should be more practical and should be connected with the real things of life. I would like to ask in what way some of the quantitative experiments in physics are any more practical than some of my algebraic equations or propositions in geometry? If the object of the proposed movement is to connect the work of the school more closely with life, I fail to see how these physics experiments will assist the movement. What I know of trolley cars, locomotives or stationary engines I did not learn in the class room.

A motion that the president appoint a committee to consider the question of laboratory work in physics and mathematics for the high schools was passed. President Petterson later appointed the following persons to serve on this committee:

Miss Mabel Sykes, South Chicago High School, chairman.

Mr. H. B. Loomis, Medill High School.

Mrs. G. R. Page, Lake View High School.

The next meeting of the association occurred March 3, 1903, and was addressed by Dr. J. W. A. Young, of the University of Chicago, on "What Is the Laboratory Method?" This paper will be published later.

Mathematical Supplement
of
School Science

VOL. I. JUNE, 1903. No. 2.

EDITORIAL.

A Working Basis for Correlation.

When the ambitious young mathematics teacher gets the problems of his immediate situation sufficiently well in hand to make it possible for him to enter upon a wider study of the problems of his vocation, one of the earliest lessons impressed upon him by a study of the practice of his professional colleagues is the importance to the mathematics teacher of open-mindedness to progress. Nor can he be long in discriminating between that sort of openmindedness which manifests itself in a willingness to give theoretical assent to doctrines and principles paraded in teachers' conventions and that more practical and more serviceable sort of openmindedness which shows itself in a willingness to try out such doctrines as commend themselves to one's approval. He soon learns that the only sort of openmindedness which can be of any consequence for progress is the sort which eventuates in practice.

Keenly alive to the great difference between what mathematics teaching is and what it should be, and burning with a zeal to bring his own practice nearer to the goal than is the common run of such teaching, the enthusiasm of our youthful aspirant too frequently leads him to place too low an estimate on the importance of "taking things as he finds them" as his starting point for advance. Perhaps more really meritorious reforms in teaching have suffered temporary or permanent shipwreck through summarily breaking contact with current conditions than through all other influences combined. This is a lesson the true friends of correlated mathematics teaching can ill afford to ignore. It should be

accepted as an axiom of the history of progress in mathematics teaching that present needs and the present school situation are the points of departure for all future pedagogical advance. Accordingly, the following suggestions for taking a step in the direction of correlating the cognate lines of study in the high school are proposed as workable principles under present systems, while at the same time they embody a response to current pedagogical needs.

Stated from the mathematical point of view the practical problem of correlation consists in the unification in teaching of the mathematical subjects and the correlation of this unity of subjects with physics and with other metrical sciences. Stated from the physical side, it consists in such an organization of the subject-matter of courses as will enable the student to lay the mathematical foundations and to put in the necessary stiffening braces and tie-beams in course of the erection of his edifice of physical truth. It is, moreover, clear that from whatever side we approach the problem of unifying subjects, now separate and distinct, we must first seek the principle, or the principles, in accordance with which the unification may be rationally effected. The following paragraphs suggest a basis upon which it is believed that algebra and geometry in the high school may be correlated in teaching into a rational unity.

Furthermore, whatever body of truth may from time to time be selected as the source from which are to be drawn the principles and ideas to be taught, it must always be possible to state the results of teaching in terms of mathematics, and of physics as well as of mental growth, else we shall find much of our teaching at loose ends and of no determinate value in any direction. While the inventory-taking *per se* may be of no great pedagogical import, still to avoid unnecessary repetition and unprofitable doubling back upon our own track we find the inventory-taking an indispensable step in the teaching process.

From the mathematical point of view the high school must furnish the student such particular applications and such informal and formal proofs as will enable him clearly to apprehend and intelligently to use the trunk principles of mathematics which are common to algebra and geometry. A parallel arrangement of the most important of these principles is the following:

GEOMETRY.	ALGEBRA.
Fundamental operations, (geometrically performed)	Fundamental operations, (algebraically performed)
Congruency,	The equation,
Vector magnitudes,	Signed numbers,
Inequality (how established?)	Inequalities,
Arcual measurement of angles,	Functionality,
Parallelism,	Simultaneity,
Similarity,	Proportionality, Variation,
Powers and roots,	Theory of exponents,
Analysis of plane figures into triangles,	Factorization of quadratic expressions,
Space figures analyzed into their plane elements,	Factorization of cubics, quartics, etc.,
Regular figures,	Symmetrical expressions,
Circle and round bodies.	Theory of limits.

Substantially all the topics of algebra and geometry fall at once as special applications under these central principles. It can scarcely be denied that a study of algebra and geometry which enables the student to penetrate to these core principles, to seize and to use them skilfully would result in much greater benefit to the student educationally than does the prevalent method of studying each proposition, or topic, as a separate link in a logical chain of thought; for power, in the last analysis, means the ability to discern, to grasp and to apply organizing principles. After the student actually grasps these few principles and senses their significance with some measure of adequacy, the vast bulk of exercises and seemingly distinct propositions in both subjects may be set as exercises for him. This is the true place of the *heuristic* method in mathematics. After some such fashion as this algebra and geometry will readily adapt themselves in a high degree to inventional treatment.

Under systems of school organization where teachers deem it unwise to deviate far from the body of subject-matter of algebra and geometry as outlined in the standard texts, or in stipulated outlines, it is believed that a presentation of this subject-matter with reference to the principles suggested above will be found to be a distinct improvement upon the customary practice.

A MORE RADICAL AND BETTER PLAN.

In the writer's opinion, however, a course in high school mathematics arranged somewhat after the following plan would be a decided improvement over even modified current courses. The proposed course provides for only three years' work and contemplates that it be given the first, second and fourth years of the high school period as this agrees with the program plans of most high schools at the present time.

FIRST YEAR'S WORK.

1. *Short course in constructive geometry.* (three months).

Subject-matter: Common experiences, architectural and ornamental designs and patterns, elements of machine design, surveying. (See Morris' Practical Pl. and Sph. Geometry, Longman's.)

2. *Short course in physics.* (three months).

Subject-matter: Simple machines, force problems and such other subjects as have a distinct leaning toward the metrical side and are capable of geometric, or algebraic treatment, leading to solutions in the form of laws expressible in geometric or algebraic language. (See Ball's Experimental Mechanics, Macmillan & Co.)

3. *Short course in the geometry and algebra of the field and laboratory.* (three months).

Subject-matter: Metrical, experimental and demonstrative proofs of such geometric and algebraic principles as are prepared for by the foregoing courses and such as are required for the second year's work in physics.

The work for the year would include a pretty complete informal treatment of the first five topics in the foregoing enumeration of principles.

SECOND YEAR'S WORK.

4. *Correlated physics, algebra and geometry.* (Second year's work).

Subject-matter: Building on the first year's work, this course should continue under the general guidance of the central principles enumerated above with less emphasis upon geometrical aspects and more upon physical and algebraic phases of science study. The role of geometry through this year should be that of a means (1) to the expression and solution of physical and simple

mechanical problems and (2) to the graphical portraiture of algebraic notions and processes. The algebra should (1) connect with the students' knowledge of arithmetic under the form of generalized number, should (2) come into the physics as a means of exact description and record of physical laws and of treating these laws and (3) its principles should parallel those of geometry while its more perfect notation should give sharpness and completeness to geometric notions.

Informal methods of establishing truths should still be given considerable prominence through the work of this year, but formal methods of demonstration should also be given much attention. Definitions should be sparingly used.

In connection with the theory of limits some of the elementary notions of the calculus should be introduced and used and with the study of similarity the ideas of trigonometry, without much terminology, should be used.

THIRD YEAR'S WORK.

5. *Quantitative physics and chemistry.* (third year's work).

The work of the third high school year should be given to physics and chemistry with more than the usual emphasis upon the quantitative side. Opportunities to exemplify mathematical principles to students should be sought, rather than shunned, in these sciences. The science teacher should no more disdain to help the student over a mathematical difficulty brought out by the science work, than should the mathematics teacher refuse to assist in clearing up a physics problem which creates trouble for the student in mathematics. In this course the needs of science should give trend to the work.

FOURTH YEAR'S WORK.

6. *Correlated mathematics and science.* (fourth year's work).

Subject-matter: In this course solid and spherical geometry should be completed, and considerable plane trigonometry, graphical representation and elementary calculus should be learned. It is not intended that the trigonometry and calculus should be studied as set topics; but rather that the ideas and methods of these subjects should be used wherever they offer advantages in point of simplicity or clearness over the methods of the elementary sub-

jects. With the space geometry should be given some geometrical drawing of the orthogonal projections upon three mutually perpendicular planes of the solids studied and of sections of them. These drawings should be made in shade and shadow as in descriptive geometry. Such work as this should begin earlier than here, but it should constitute a feature of the work of this year.

Organizing principles: The dominating idea of this course should be the unifying of the students' experiences up to date. The work should take its origin largely from the data and inferences furnished by former experiences and experiments, but theoretical consequences may here be pursued further into the realm of abstraction that has hitherto been desirable. The course should, in some sense, round out the student's high school education in mathematics, but so far from leaving him with the feeling that he has completed pretty much all there is to be known in mathematics, it should open to him the possibilities of further study which lie beyond him. The writer believes the latter aim is of more consequence than the former, even though the student's scholastic career must end with the high school.

Finally, the propositions of this plan are intended only as a provisional scheme for work, and they will have accomplished their mission if they shall succeed in calling out from our readers something better to supplant it. It is only claimed that a reorganization of high school mathematics on the basis here suggested would result in an outcome which would be much better than the common outcome both practically and educationally. It is hoped that the plan may succeed in provoking a much better scheme which we shall be glad to dignify by publication if the author of such scheme will favor us with a copy of it.

SUGGESTIONS ON THE INTRODUCTION OF A COURSE OF CORRELATED ALGEBRA, GEOMETRY AND PHYSICS.

BY MISS EDITH LONG.

Lincoln (Neb) High School.

In compiling a course of the three subjects, physics, algebra and geometry, usually, so far as a school curriculum is concerned,

kept entirely separate, it is difficult to decide on the order of introduction, and having settled that, it is still a problem how so to weld the three together that each may gain strength from the others.

Two methods of introduction present themselves. One, the simultaneous introduction of the three, each carried along as needed in the development, not taught on alternate days nor necessarily all on the same day, but each taken up as it may elucidate or complete the other. Second, the introduction of one of the subjects, and, after a fair comprehension of this is obtained, the others in succession until all three are carried on in the manner and with the same end in view as by the first method. The choice of plan depends upon whether the same teacher has charge of both physics and mathematics. If he has, then he can introduce just such parts of either as he needs and develop both at the same time. If he has not, then there must be on the part of the child a larger stock of knowledge to draw from. For an ideal correlation, the smaller schools with but one teacher for both mathematics and physics have an advantage over the larger schools with rigid lines drawn on department work. In the latter, the second plan suggested has its advantages.

The next question that arises is, if one subject is introduced first, which should it be? For my own part, I should say decidedly that subject which appeals most to the child. Physics, dealing as it does with the definite, concrete, sensuous facts is plainly that subject. Abstract reasoning is not in the make up of child nature, then why attempt to force it upon him? Why not so gradually lead him through the concrete which he does enjoy into the more and more abstract that he may little by little come to enjoy that also? True it is that nearly all courses of study and all textbooks are built upon precisely the opposite idea. Algebra, the most abstract of all, is introduced first, and in the algebras, especially in equation work, every suggestion that the equation has its origin in practical experience is studiously avoided. The student is made to wade through pages of dry mechanical examples. When the application is reached the student must begin all over again: for even if he sympathizes with *A's* and *B's* struggle in money affairs that he knows nothing about or should care to

empty cisterns by sets of pipes of different sizes, the expressions have such a different setting that he does not recognize them. Why not first give him experiences to draw from and then build upon them? Is there a better place to gain these experiences than in the laboratory?

Acting upon the aims suggested by these questions the mathematics teachers of our school have this year fortunately been able to have a semester of experimental physics preceding the mathematics proper. The expression mathematics proper is used because during the aforesaid semester the mathematics has been by no means neglected. So far as possible, the student has been called upon to calculate the results of his measurements and much of the work, when mathematics is begun, is but to formulate in algebraic language the knowledge already gained. This is decidedly true in the case of positive and negative quantities, addition, subtraction, multiplication, division, simple ratio and proportion and equation work. The student has seen all kinds of forces acting against one another; he has measured many and the results of their combined action. He has also watched forces acting at angles to one another and noted the results so that even additions of vectors are not a surprise to him. From his own use of the balances he has a very fair comprehension of the comparative weights of many kinds of substances and he discusses intelligently problems leading to equations which may arise from these comparisons. Again when the discussion comes up to bring out the notion of geometric solids, planes, lines and points you find him on safe ground, for he has seen and even made for himself in the laboratory a host of crystal forms and he has handled apparatus of every shape imaginable. Above all it is surprising to note the self assurance with which he takes up new subjects or goes on with the investigation of the old. This arises from his self dependence in the laboratory. As has been said, the introduction of mathematics under these conditions is but the clothing of fairly well grounded information in algebraic and geometric language.

A serious problem to those who would adopt correlation is what shall be the amount of time devoted to each study and how shall the specific time be distributed? Shall the subjects be regularly taken up on alternate days or must each come up every

day? Now it seems to me that any forced plan would be disastrous to the work. Many have tried to teach algebra and geometry on alternate days, religiously assigning the one and the other regardless of whether there is any connection between the two. The result is, as might be looked for, a loss of interest in both subjects. To teach algebra, geometry and physics together, as has been said, there must be a real correlation, a welding of the three. If the same teacher is well informed in each branch and has in mind the equally successful teaching of each, and the thorough understanding of each on the part of the student there will be no danger of neglect of any one. He will turn as naturally to the physics when he needs an illustration for his algebra as he will turn to the algebra when he wishes to generalize the laws of the particular cases which the student has drawn from his laboratory experiment. The only chance of failure to correlate mathematics and physics lies in the lack of knowledge of either subject on the part of the instructor or his inability to pull away from the monotonous drilling in mechanical operations in algebra. Either of these faults will prove certain failure.

As a general plan for a course of study the following is suggested as having been tried and, it is believed, successfully. That each subject in algebra be introduced through examples taken from physics and geometry, and the explanation of the meaning given through these. Then the more important principles of the solution of examples will be understood before the solution is attempted. This is certainly true in all the subjects mentioned above together with simultaneous and quadratic equations.

Any purely abstract mathematical subjects should be left until late in the course when the student has gradually been lead to deal with such work.

The question is often asked at what period shall the student be expected to do abstract reasoning. At no set time but gradually should he pass from concrete into abstract thought. From the very first the work is not complete until it is generalized; for the ability to generalize intelligently marks the success of teaching on any plan.

WHAT IS THE LABORATORY METHOD?(1)

BY J. W. A. YOUNG,

Assistant Professor of the Pedagogy of Mathematics, University of Chicago.

I have given as caption of what I am about to say, a question I am not prepared to answer.

We have recently been discussing here in Chicago various phases of the teaching of mathematics at various times and in various bodies; the term "Laboratory Method" has been used, but not (to my recollection) strictly defined. It has rather been used, I take it, in accordance with that sound pedagogic dictum, *Ideas first, definitions afterward,* as a convenient term for what we are trying to work out; a symbol—*the x-method*—whose meaning we are engaged in fixing.[2]

In my mind, this x-method is somewhat vaguely outlined by various characteristics, no one of which constitutes the method or is peculiar to it.

I have thought that a brief enumeration of these characteristics might make a useful basis for discussion. Perhaps by the free addition of other characteristics and the elimination of any which are improperly here, the discussion may give us a somewhat closer approximation to a common understanding of what we mean in saying "*the* laboratory method."

For brevity, I wish at the outset to prefix what I shall say with a large *"It seems to me,"* and to disclaim any dogmatic intention in adopting a more or less dogmatic form for clearness and brevity.

CHARACTERISTICS.

I. A DESIRE TO IMPROVE.

This is the chief single characteristic—the perception by each one of us that his own teaching *can* be improved, and the active purpose to do so.

The x-method welcomes and assimilates *whatever* is an im-

(1) Address delivered before the Mathematics Section of the Central Association of Science and Mathematics Teachers, at Armour Institute of Technology, Chicago, April 11, 1903.

(2) "A laboratory system involving a synthesis and development of the best pedagogic methods at present in use in mathematics and the physical sciences." *Moore*, On the Foundations of Mathematics, Presidential address; *Bull. Am. Math. Soc.*, 1903, p. 424; also *Science*, 1903, p. 416; also being published in *School Science*.

provement. It recognizes, also, that the determination whether or not any proposed change is an improvement must take into account the purpose of the teaching of mathematics as well as the personality of the teacher and the conditions under which he works.

We are assuming as basis of our discussions a clearly formulated idea of the function of mathematics in secondary education; of the purpose and value of the teaching of mathematics. This is too large a topic to open today, and yet its careful consideration must both logically and actually, precede consideration of methods. No teacher of mathematics can do his best work who has not the ultimate purpose for which he is teaching mathematics, constantly, consciously, in mind, as the final arbiter of *what* he is to teach, and *how* he is to teach it.

Methods are but *means;* the utility of the means can be judged only from the point of view of the *end.* .

The x-method fosters whatever hastens progress toward the *end,* and accepts nothing else.

It frowns upon *fads;* that is, the exaltation of the means into *ends in themselves.*

All of us are conditioned by our environment. The work we have to do is more or less minutely mapped out for us by curricula, by examination standards, by public opinion in the profession and in the community.

While this frees none of us from the obligation of forming his own clear ideas and ideals of the function of his subject, it may impose on him some specific things to be done. He has certain things to teach, *in fact,* whether these topics are selected by himself in conformity with his own ideals, or by others according to their ideas.

In either case the ultimate end is resolved into a series of mediate ends—the teaching of specific subjects and topics. What should be the teachers attitude as to method under these (not ideal, but very real) conditions? Evidently his task is to lead his pupils to intelligent mastery of the things taught, and withal to develop thought-power which shall be available later. Let him then take as his watchword, dominating all methods, *clearness;*

use the x-method, the y-method or the z-method when (and only when) they aid in attaining clearness.

These things are all truisms—fundamental postulates—but I hope a mathematical audience will pardon the formulation of some fundamental postulates.

In entering into details, I must confine myself very strictly and briefly to points that have been mentioned in the present series of discussions on the laboratory method. Such mention has also been the sufficient credential for the admission of any point.

It has not seemed desirable to attempt to avoid redundancies, the implication of one characteristic by another, or the more or less complete overlapping of topics; the various heads have not even been scrutinized as to compatibility. No attempt has been made to determine or indicate their relative weight or importance.

II. INTEREST.

That we work best only when interested, is generally believed.

A common figure for this is that of food. To be most effectively nutritious, food must be palatable. This has been appreciated for centuries, still the Nobel prize was awarded in 1901 (3) for researches definitely establishing close connection between appetite and digestion, and showing that for best results food should be eaten with interest and enjoyment.

So with us, in mathematics. We have long known that to be most nutritious, mathematics must be palatable, but our present discussions are evidently stimulated by a more vivid consciousness of this truth. We are making a stronger effort to awaken effectively the appetite so essential to mathematical digestion.

III. CHARACTERISTICS RELATING TO SUBJECT MATTER.

1. CORRELATION OF SUBJECTS.

a. The mathematical subjects among themselves. Geometry and algebra (involving and illuminating arithmetic constantly, and developing the elements of trigonometry on occasion) should be taught side by side, and not separately. No "water-tight compartments."

b. Mathematics with physics. Let physics be taught simul-

(3) The *Hospital*, March, 1903.

taneously with mathematics, so that the demand for mathematical treatment of questions may arise in physics, and so that physics may furnish the opportunity for immediate application of mathematical theory.

2. AMALGAMATION OF MATHEMATICS AND PHYSICS.

Teach mathematics no longer; teach physics no longer. Take what has hitherto been taught under these titles, and with suitable eliminations and additions, amalgamate the whole mass into a homeogeneous whole.

This is questionable (*a*) as to *feasibility*. Can the dynamic and static phases be disregarded? Can the making of experiments, the actual manipulation and the quiet thinking, the working out of the theoretic relations be combined? (*b*) as to *desirability*. "I want my roast beef and pudding *correlated,* but I don't want them *amalgamated.*"

IV. CHARACTERISTICS RELATING TO MODE OF INSTRUCTION.

1. PROCEED FROM THE CONCRETE TO THE ABSTRACT.

Let experiments be freely used (physical, metrical, graphic, numerical). Let the general definitions and theorems be suggested by concrete special cases and be developed out of them by the pupils if possible. Don't banish the *abstract* from the teaching of mathematics, but change it from a noun (or adjective) to a verb. Never teach *abstract* mathematics but only *abstracted* mathematics and as far as possible let the class abstract its own mathematics. Such work, studying general laws through particular instances, is analogous to that of the physical *laboratory.*

REMARK. The term *concrete* must be taken in a broad sense. *What* is concrete depends upon circumstances. The physically concrete things, the things that can be handled, should be utilized in the mathematical instruction to a much greater extent than heretofore. But other things are concrete also. For example, in algebra much of arithmetic is concrete. In college mathematics (and there is no reason why the main thoughts of what has been said should not be applied in college mathematics) the results of the previous mathematics which have been well grasped would all come under the head *concrete,* used broadly. With such a use of the term, is there any upper limit to the applicability

of the dictum: *Proceed from the concrete to the abstract; from the particular to the general?*

The need of more stress on the concrete, the physical side, is being emphasized in Germany also. In a recent address Professor Klein, of Göttingen, said:

"No teaching in gymnasium and *real schule* is so difficult as that of mathematics, since the large majority of the pupils is decidedly indisposed to allow itself to be harnessed in the rigid framework of logical conclusions. The interest of young people is much more easily won if one sets out from sense-objects and gradually leads on to abstract formulations. It is therefore psychologically quite correct to follow this path. This course commends itself none the less if we inquire concerning the real goal of instruction in mathematics. Sharpening the understanding was formerly regarded as the end. Another chief end is: To make the conviction grow that *correct thought on the foundation of correct premises gives mastery of the external world.* To do this, attention must be directed to the external world from the beginning.

"All this is certainly most true; but there lies a danger in it, and to this danger I wisn to call attention today. . . . It is possible that through the mere mass of interesting applications, the real logical training may be crippled, and under no circumstances may this happen, for then the real marrow of the whole is lost. Hence: We desire emphatically, an enlivening of instruction in mathematics by means of its applications, but we desire also that the pendulum which in earlier decades perhaps swung too far in the abstract direction, should not now swing to the other extreme, but we wish to remain in the just mean.

"To preserve the just mean is the problem and the art of the teacher, which should be advanced through an improved *preparation of teachers.*" [4]

Note.—It will be remembered that the preparation of the Prussian teachers to whom reference is made, already consists of three years of graduate study of theoretic mathematics at the University, followed by two years of practical training in the art of teaching.

Similar ideas were also presented and discussed at the *Versammlung deutscher Philologen und Schulmänner,* in Strassburg, September, 1901.([5])

2. TEACH THROUGH THE EYE:

a. By orderly arrangement (laboratory record book, geometry tablets, etc.)

b. By use of colors.

c. By careful drawing.

d. By use of squared paper.

([4]) Klein: *Ueber den Mathematischen Unterricht an den hoheren Schulen.* Jahresbericht der deutschen Mathematiker Vereinigung, 1902, pp. 128-140.

([5]) *Zeitschrift fur mathematischen und naturwissenschaftlichen Unterricht,* 1902, p. 669.

The last is specially valuable (1) to show more vividly and clearly relations running through a mass of data; (2) to give geometric form to algebraic relations.

3. TEACH EVERYTHING IN TWO WAYS:

When one form of presentation is not clear perhaps another will be. Show the different bearings and interpretations of the same truth. All this is subject to the more fundamental dictum: *Teach everything clearly.* Use different ways of expressing the same thought whenever one illustrates and illuminates the other. Due caution to avoid confusion is, of course, needed.

4. WORK WITH A LARGE BODY OF AXIOMS. Assume many things which might be proved. Defer the more philosophic questions.

5. ACCEPT AND ENCOURAGE PROOFS BASED ON INTUITION; PROOFS BASED ON MEASUREMENT.

This is a reaction from the attempt to deny the validity of such proofs. It is not meant that there should not be developed from them (by generalization) the customary abstract proofs, nor that we should overlook one of the special functions of mathematics —training in precision—and the need of such training.

6. LESS STRESS ON EXHIBITION OF WORK done outside; more time given to actual work in the class, thus making the classroom a working place, a *laboratory.*

This does not mean that there is to be *no* exhibition of work.

Societies of learned mathematicians exist, one of whose main purposes is the exhibition of work, and the mathematician will surely not deny to young pupils the benefit and pleasure which he does not forego himself.

It does mean that the teacher helps the pupil to *do* work, rather than *examines* him to find whether or not he has done an allotted task.

7. DEVELOP NEW MATTER INDUCTIVELY WITH THE CLASS. The pupil should seldom, if ever, be allowed to struggle through new theories and processes alone for the first time. Let the teacher develop the theory, with the text in the background for reference.

See President Hadley, Meaning and Purpose of Secondary Education. *School Review*, December, 1902, p. 738.

Such work also bears considerable analogy to that of a physical laboratory.

8. TRAIN ENGINEERS AS WELL AS PROFESSORS. That is, do not proceed on the assumption that every pupil is to be a professional mathematician. A better formulation would be: *Train human beings;* prepare good citizens, equipped with a well rounded education, and not specialists of any sort. Let the university and the technical school train the professor and the engineer.

9. NO DAILY ALLOTMENT OF WORK, the same for all each day. Rather a general allotment, each one working on at the rate best suited to his strength (laboratory idea).

10. PUPILS WORK INDEPENDENTLY. The instructor is a friend, co-worker, not a task master.

11. PUPILS WORK TOGETHER. Mutual assistance and dissemination of results encouraged. This does not contravene what all will agree is one of the chief ends of the teaching of mathematics, viz., to teach the pupil to *think for himself.* We all get suggestions from others, to the benefit rather than impairment of our independence.

12. TWO-HOUR SESSIONS. If the classroom is the workshop and nearly all of the work is done there, a one-hour session is inadequate.

V. SUMMARY.

What are the chief results of what has been said?

1. Close correlation of physics, arithmetic, geometry, algebra, trigonometry.
2. In teaching pass from concrete to abstract.

V. WHAT CAN TEACHERS DO NOW TO ATTAIN THESE ENDS?

This question has been answered so well and in so stimulating a way by the previous papers and by the report of the committee that it would be superfluous at this juncture to say more.

With the motto: *Evolution, not revolution,* let the teacher adopt whatever is good, making some improvement each year, each day, and the evolution will go forward wonderfully well.

ELEMENTARY MATHEMATICS.*

BY ELIAKIM HASTINGS MOORE.

Professor and Head of Mathematical Department, University of Chicago.

(*Continued from page 28.*)

This separation between pure mathematics and applied mathematics is grievous even in the domain of elementary mathematics. In witness, in the first place: The workers in physics, chemistry, and engineering need more practical mathematics; and numerous textbooks, in particular, on calculus, have recently been written from the point of view of these allied subjects. I refer to the works by Nernst and Schoenflies,[13] Lorentz,[14] Perry,[15] and Mellor,[16] and to a book on the very elements of mathematics now in preparation by Oliver Lodge.

In the second place, I dare say you are all familiar with the surprisingly vigorous and effective agitation with respect to the teaching of elementary mathematics which is at present in progress in England, largely under the direction of John Perry, professor of Mechanics and Mathematics in the Royal College of Science, London, and chairman of the Board of Examiners of the Board of Education in the subjects of engineering, including practical plane and solid geometry, applied mechanics, practical mathematics, in addition to more technical subjects, and in this capacity in charge of the education of some hundred thousand apprentices in English night schools. The section on Education of the British Association had its first session at the Glasgow meeting, 1901, and the session was devoted to the consideration, in connection with the section on Mathematics and Physics, of the question of the pedagogy of mathematics, and Perry opened the discussion by

---

* Presidential address, *On the Foundations of Mathematics*, delivered before the American Mathematical Society at its ninth annual meeting, December 29, 1902.

[13] Nenrst und Schoenflies: Einführung in die Mathematische Behandlung der Naturwissenschaften (Munich and Leipzig, 1895); the basis of Young and Line Naturwissenschaften (Munich and Leipsic, 1895); the basis of Young and Line barger's Elements of Differential and Integral Calculus (New York, 1900).

[14] Lorentz: Lehrbuch der Differential und Integralrechnung (Leipsic).

[15] Perry: Calculus for Engineers (Second Edition, London, E. Arnold, 1897); German translation by Fricke (Teubner, 1902). Cf. also the citations given later on.

[16] Mellor: Higher Mathematics for Students of Chemistry and Physics, with special reference to Practical Work. (Longmans, Green & Co., 1902, pp. xxi+543)

a paper on "The Teaching of Mathematics". A strong committee under the chairmanship of Professor Forsyth of Cambridge was appointed "to report upon improvements that might be effected in the teaching of mathematics, in the first instance, in the teaching of elementary mathematics, and upon such means as they think likely to effect such improvements". The paper of Perry, with the discussion of the subject at Glasgow and additions including the report of the committee as presented to the British Association at its Belfast meeting, September, 1902, are collected in a small volume, "*Discussion on the Teaching of Mathematics,*"[17] edited by Professor Perry (Macmillan; second edition, 1902).

One should consult the books of Perry, *Practical Mathematics,*[18] *Applied Mechanics,*[19] *Calculus for Engineers,*[20] and *England's Neglect of Science,*[21] and his address[22] on *The Education of Engineers,*—and furthermore the files from 1899 on—of the English journals, *Nature, School World, Journal of Education,* and *Mathematical Gazette.*

One important purpose of the English agitation is to relieve the English secondary school teachers from the burden of a too precise examination system, imposed by the great examining bodies; in particular, to relieve them from the need of retaining Euclid as the sole authority in geometry, at any rate with respect to the sequence of propositions. Similar efforts made in England about thirty years ago were unsuccessful. Apparently the forces operating since that time have just now broken forth into successful activity; for the report of the British Association committee was distinctly favorable, in a conservative sense, to the idea of reform, and already noteworthy initial changes have been made in the regulations for the secondary examinations by the examination syndicates of the universities of Oxford, Cambridge, and London.

---

[17] Cf. also Report on the Teaching to Elementary Mathematics, issued by the Mathematical Association (G. Bell & Sons, London, 1902).

[18] Published for the Board of Education by Eyre and Spottiswoode. (London, 1899).

[19] D. Van Nostrand Co., New York, 1898.

[20] Second edition, London, E. Arnold, 1897.

[21] T. Fisher Unwin, London, 1900.

[22] In opening the discussion of the sections on Engineering and on Education at the Belfast, 1902, meeting of the British Association. Published in Science, Nov. 14, 1902.

The reader will find the literature of this English movement very interesting and suggestive. For instance, in a letter to *Nature* (vol. 65, p. 484; March 27, 1902) Perry mildly apologizes for having to do with the movement whose immediate results are likely to be merely slight reforms, instead of the thoroughgoing reforms called for in his pronouncements and justified by his marked success during over twenty years as a teacher of practical mathematics. He asserts that the orthodox logical sequence in mathematics is not the only possible one; that on the contrary a more logical sequence than the orthodox one (because one more possible of comprehension by the students) is based upon the notions underlying the infinitesimal calculus taken as axioms: for instance, that a map may be drawn to scale; the notions underlying the many uses of squared paper; that decimals may be dealt with as ordinary numbers. He asserts as essential that the boy should be *familiar* (by way of experiment, illustration, measurement, and by every possible means) with the ideas to which he applies his logic; and moreover that he should be thoroughly *interested* in the subject studied; and he closes with this peroration:

> "'Great God! I'd rather be
> A pagan, suckled in a creed outworn.'

I would rather be utterly ignorant of all the wonderful literature and science of the last twenty-four centuries, even of the wonderful achievements of the last fifty years, than not to have the sense that our whole system of so-called education is as degrading to literature and philosophy as it is to English boys and men."

As a pure mathematician, I hold as the most important suggestion of the English movement the suggestion of Perry's, just cited,—that by emphasizing steadily the practical sides of mathematics, that is, arithmetic computations, mechanical drawing and graphical methods generally, in continuous relation with problems of physics and chemistry and engineering, it would be possible to give very young students a great body of the essential notions of trigonometry, analytic geometry, and the calculus. This is accomplished on the one hand by the increase of attention and comprehension obtained by connecting the abstract mathematics with subjects which are naturally of interest to the boy, so that, for instance, all the results obtained by theoretic process are capable of

check by laboratory process, and on the other hand by a diminution of emphasis on the systematic and formal sides of the instruction in mathematics. Undoubtedly many mathematicians will feel that this decrease of emphasis will result in much if not irreparable injury to the interests of mathematics. But I am inclined to think that the mathematician with the catholic attitude of an adherent of science in general (and at any rate with respect to the problems of the pedagogy of elementary mathematics there is no other rational attitude) will see that the boy will be learning to make practical use in his scientific investigations, to be sure, in a naive and elementary way, of the finest mathematical tools which the centuries have forged, that under skilful guidance he will learn to be interested not merely in the achievements of the tools but in the theory of the tools themselves, and that thus he will ultimately have a feeling towards his mathematics extremely different from that which is now met with only too frequently,—a feeling that mathematics is indeed itself a fundamental reality of the domain of thought, and not merely a matter of symbols and arbitrary rules and conventions.

THE AMERICAN MATHEMATICAL SOCIETY.

The American Mathematical Society has, naturally, interested itself chiefly in promoting the interests of research in mathematics. It has, however, recognized that those interests are closely bound up with the interests of education in mathematics. I refer in particular to the valuable work done by the committee appointed, with the authorization of the Council, by the Chicago Section of the Society, to represent mathematics in connection with Dr. Nightingale's committee of 1899 of the National Educational Association in the formulation of standard curricula for high schools and academies, and to the fact that two committees are now at work, one appointed in December, 1901, by the Chicago Section, to formulate the desirable conditions for the granting, by institutions of the Mississippi valley, of the degree of Master of Arts for work in mathematics, and the other appointed by the Society at its last summer meeting to coöperate with similar committees of the National Educational Association and of the Society for the Promotion of Engineering Education, in formulating

standard definitions of requirements in mathematical subjects for admisson to colleges and technological schools; and furthermore I refer to the fact that (although not formally) the Society has made a valuable contribution to the interests of secondary education in that the College Entrance Examination Board has as its Secretary the principal founder of the Society. I have accordingly felt at liberty to bring to the attention of the Society these matters of the pedagogy of elementary mathematics, and I do so with the firm conviction that it would be possible for the Society, by giving still more attention to these matters, to further most effectively the highest interests of mathematics in this country.

A VISION.

AN INVITATION.

The pure mathematicians are invited to determine how mathematics is regarded by the world at large, including their colleagues of other science departments and the students of elementary mathematics, and to ask themselves whether by modification of methods and attitude they may not win for it the very high position in general esteem and appreciative interest which it assuredly deserves.

This general invitation and the preceding summary view invoke this vision of the future of elementary mathematics in this country.

THE PEDAGOGY OF ELEMENTARY MATHEMATICS.

We survey the pedagogy of elementary mathematics in the primary schools, in the secondary schools, and in the junior colleges (the lower collegiate years). It is, however, understood that there is a movement for the enlargement of the strong secondary schools, by the addition of the two years of junior college work and by the absorption of the last two or three grades of the primary schools, into institutions more of the type of the German gymnasium and the French lycée;[23] in favor of this movement there are strong arguments, and among them this, that in such institutions, especially if closely related to strong colleges or universities, the mathematical reforms may the more easily be carried out.

[23] As to the mathematics of these institutions, one may consult the book on "The Teaching of Mathematics in the Higher Schools of Prussia." (New York: Longmans, Green & Co., 1900) by Professor Young, and the article (Bulletin Amer. Math. Soc. (2) vol. 6, p. 225) by Professor Pierpont.

The fundamental problem is that of *the unification of pure and applied mathematics.* If we recognize the branching implied by the very terms "pure", "applied", we have to do with a special case of *the correlation of different subjects* of the curriculum, a central problem in the domain of pedagogy from the time of Herbart on. In this case the fundamental solution is to be found rather by way of indirection,—by arranging the curriculum so that throughout the domain of elementary mathematics the branching be not recognized.

THE PRIMARY SCHOOLS.

Would it not be possible for the children in the grades to be trained in power of observation and experiment and reflection and deduction, so that always their mathematics should be directly connected with matters of thoroughly concrete character? The response is immediate that this is being done today in the kindergartens and in the better elementary schools. I understand that serious difficulties arise with children of from nine to twelve years of age, who are no longer contented with the simple concrete methods of earlier years and who, nevertheless, are unable to appreciate the more abstract methods of the later years. These difficulties, some say, are to be met by allowing the mathematics to enter only implicitly in connection with the other subjects of the curriculum. But rather the material and methods of the mathematics should be enriched and vitalized. In particular, the grade teachers must make wiser use of the foundations furnished by the kindergarten. The drawing and the paper folding should lead on directly to systematic study of intuitional geometry, with simple exercises in geometrical reasoning.[24] The children are to be taught to represent, according to the usual conventions, various familiar and interesting phenomena. The geometry must be closely connected with the numerical and literal arithmetic. The cross-grooved tables of the kindergarten furnish an especially important type of connection, viz., a conventional graphical depiction of any phenomenon in which one magnitude depends upon another. These tables and the similar cross-section blackboards and paper must enter largely into all the mathematics of the

[24] Here I refer to the very suggestive paper of Benchara Branford, entitled "Measurement and Simple Surveying. An Experiment in the Teaching of Elementary Geometry" to a small class of beginners of about ten years of age (Journal of Education London—the first part appearing in the number for August, 1899.)

grades, and to study the properties of the phenomena in the pictures; to know, for example, what concrete meaning attaches to the fact that a graph curve at a certain point is going down or going up or is horizontal. Thus the problems of percentage,—interest, etc.,—have their depiction in straight line or broken line graphs.

(*To be continued.*)

WHAT HIGH SCHOOL PHYSICS SHOULD REQUIRE FROM ALGEBRA AND GEOMETRY.

BY F. L. BISHOP.

Bradley Polytechnic Institute.

"Education," says Lyman Abbott, "should furnish the student with two things for his work in life; facts or information and power, or ability to use these facts."

The study of algebra and geometry has furnished the student with an abundance of facts and information, concerning algebraic expressions and geometrical propositions, but it has failed almost utterly to supply him with the power to apply these facts to the solution of the problems found in elementary physics.

It is not the intention to enter into a general discussion in this paper as to why the study of these subjects has failed to furnish this power, but only to outline some of the facts of algebra and geometry to show where more power is needed.

These requirements with a brief statement as to why they exist may be stated as follows:

1st. Ability to solve an equation no matter what letters may be used, *i. e.*, $x = \frac{1}{2} y z^2$ or $s = \frac{1}{2} a t^2$ or $v = \frac{1}{2} m t^2$ all of which are forms of the same equation.

One of the most common and troublesome difficulties is that a student can work all sorts of expressions between x, y, z, a, b, c, but when given the same expressions between s, a, t, v, etc., he stumbles at once. Of course, he ought to see that there is no difference between the expressions but the fact remains that for some reason he fails more or less completely to grasp the relation This is probably due to the fact that the subject and terms used

are new and perhaps he has seen but few expressions which did not contain the letters x, y, z, a, b, c in a more or less definite relation. That this is possible is easily seen from the following: I take an algebra which is quite extensively used and, opening it at random, several times count all the expressions found on these pages. There are 284 and all but *four* contain only the letters x, y, z, a, b, c.

2nd. Every term and algebraic expression used in physics should be found in the problems of algebra.

All laws of physics can be expressed in the form of algebraic equations, though to the elementary student this is not easy. He can learn the law, memorize the equation and from memory know which equation goes with a certain law and still fail altogether to see the reasonableness of the thing. This arises undoubtedly from the following: I take the same algebra previously referred to and find there are 2,907 problems and expressions through quadratics, which are to be worked by the student. Of these only 25 refer in any way to the subject of physics and these refer almost entirely to uniform motion. So the student has to learn after he enters physics that algebra has any relation to the new subject.

By the introduction of a large number of physical problems the student sees at once the application of the algebra and at the same times becomes familiar with the words and terms used in physics.

3rd. The use of the graph wherever and whenever possible and the derivation of the equation when the graph is a straight line.

Experimental results in physics are always expressed in the form of a graph if possible. The law connecting the results is often easily obtained from the graph. It is also of great assistance to the student to be able to construct the graph of an equation which represents a fundamental law as this shows him at once the relation existing between the quantities. The use of the cross section paper in this and other ways should be as extensive as possible.

4th. Ratio and proportion to be studied as early as possible—at least before the theory of quadratic equations.

A great many laws in physics are stated in the form of a ratio, *i. e.,* Boyle's law is often stated as follows: "At a constant

temperature the volume of a given mass of gas varies inversely as the pressure to which it is subjected."

In fact there is probably no other mathematical expression so extensively, used in physics as that one quantity *varies directly* or *varies inversely* as another. The subject of variation, which is so extensively employed is not usually taught in the first year of algebra.

5th. A knowledge of the simple trigonometric functions, sine, cosine and tangent.

Many problems in physics require the solution of triangles. Without trigonometry these problems must be limited to right triangles or they involve the student in an almost endless amount of computation which makes the problem so complicated that the physical principles involved are over-shadowed.

With this knowledge of trigonometry, physics and geometry are much more interesting, the field is broadened and the student gains at once a working knowledge of the subjects.

6th. Every physical phenomenon which involves a *directed quantity* should be made the basis of a large number of practical problems in geometry.

Prof. John Perry says: "The basis of all applications of mathematics in physics and engineering is the fact that any physical phenomenon which is *directional* (such as a force, a velocity, an acceleration, a stress, the flow of a fluid, etc.) may be represented in a most perfect manner by a straight line."

Although this great truth has been known for years, we have yet to see the geometry which introduces these problems.

7th. Measurement of lines, areas and volumes in both the metric and English systems.

The student of geometry deals with the lines, angles, areas and volumes as if they were exact and could be measured exactly. When he enters the physics laboratory he is apt to do one of two things; either work very hard to obtain the theoretical value which the book may give or decide that the apparatus is not good enough to give the results and accept any value as being good enough. In fact, he has no idea of the relation between the actual and theoretical values, of the quantities to be measured, or of the error which must always enter every measurement. This knowledge can be obtained only by making measurements.

8th. Experiments under measurement, force, center of mass, inclined plane, etc., which have hitherto been done almost exclusively in the physics laboratory to be done in the mathematical laboratory.

Before we can successfully teach any subject we must create an interest in the subject which by its momentum will carry the student over the first dull routine of the theory. This is best accomplished by an experiment performed by the student.

The following experiment illustrating a number of mathematical propositions and at least one great fact of physics is suggested as an example of this class of experiments.

Stick two pins through a piece of cardboard for a support, and on their points balance a carefully cut triangular piece of cardboard. Join by means of a straight line the holes made by the pin points. Balance the triangle again in such a way that the second line cuts the first. With a pin make a hole through the cardboard at the point of intersection of the two lines and pass a knotted thread through the hole. Support the triangle by the thread and explain what happens. Draw a line from each of the vertices to the pin-hole and extend it. Make a drawing of your triangle and name the parts.

9th. The use of logarithms and the slide rule as aids to computation.

In much experimental work as also in the solution of many problems there is of necessity a large amount of computation, which is often so extensive and so complicated that the student fails to grasp the fundamental principles involved. This routine work can be greatly diminished by employing modern methods of computation. For instance, few teachers know how to divide or multiply numbers or to square or extract square roots by means of the slide rule and yet the use of this great labor-saving device can be taught in a few minutes.

Most of these requirements have been in use at Bradley Institute for some time; others are being used here for the first time, but they have been tested sufficiently to justify the conviction that they are all perfectly practicable working suggestions.

It is at once evident that an instructor of mathematics must be thoroughly familiar with physics and this can be accomplished best by his teaching both subjects at the same time.

This method it is believed is a generally workable plan by which the artificial line that has so long existed between algebra, geometry and physics may be erased and the three subjects may be correlated into one fundamental science subject running through three years of the secondary school.

ALGEBRA AND GEOMETRY SHOULD BE TAUGHT SIDE BY SIDE IN THE HIGH SCHOOL.

BY MALCOLM M'NEILL.

Professor of Mathematics and Astronomy, Lake Forest University, Lake Forest, Ills.

My experience in the teaching of mathematics and science in secondary schools has been limited; but I have seen the work in a good many schools, and as a teacher in college for a number of years I have had abundant opportunity to examine the results. My remarks will be mainly confined to the subject of mathematics, but the argument can be used with almost equal force in regard to mathematical science. However, I leave that matter to those who have more experience than I have in elementary science teaching.

The college freshman must in most cases after admission to college do additional work in algebra and geometry; and it is here that I have opportunity to see the results of his preparatory work. My experience is that in a large majority of cases the student has a much firmer grasp of the principles of geometry than of algebra. While I can take for granted that he is well enough prepared in plane geometry to go on successfully with solid, I find that his knowledge of algebra is often too defective to enable him to take up profitably the work at the point to which his high school training is supposed to have carried him, and a considerable amount of valuable time must be consumed in a review of fundamentals. No doubt a part of this is due to the greater inherent difficulty of algebra for the average student; but it seems to me that the order of presentation of the subjects, which almost precludes mutual aid, is responsible for a considerable part of the trouble.

The average high school course—algebra first, then geometry,

and perhaps a brief review just before graduation after a year of rest from all mathematics—seems to me the worst possible arrangement. I am glad to say that it does not obtain in all schools. A considerable and, I trust, an increasing number teach the subjects side by side.

Experience no doubt shows that it is not very difficult to teach first and second year high school pupils a certain amount of manual dexterity in algebraic manipulation. But their minds are still too immature to gain any adequate grasp of the real science. They are not old enough to appreciate the underlying principles, and it is scarcely worth while to attempt to give a really scientific presentation of the subject at that stage of their development. The time for review at the end of the course must be given almost wholly to getting back what has been forgotten in their year or two of rest from algebra—forgotten because at the time they first studied the subject their minds were too immature for a comprehensive grasp. What has been learned as a mass of disconnected processes and more or less uncomprehended rules cannot in the nature of the case be any very vital part of their permanent mental equipment. The great majority of students evince a permanent dislike for algebra largely for the reason that their main training in the subject was given at a time when they were not sufficiently developed mentally to assimilate it properly.

If algebra and geometry are to be taught "tandem," it seems to me that the better plan is to take up geometry first and to follow with algebra; but I think the side-by-side method is still better. The young student acquires a comprehension of the subject matter and the method much more easily in geometry than he does in algebra. The former is so much more concrete and definite. Experience and sensation, especially of vision, render an aid lacking in algebra. A concrete representation of the steps of the process as well as of the conclusion is possible. This is hardly possible in algebra until there is some store of geometric facts at the command of the pupil. If he has, in addition, work in some science requiring measurement and computation from measurement, he has another great aid toward his comprehension of algebra.

Algebra may be studied entirely apart from geometry, and geometry may be learned without algebra. The Greeks developed a beautiful system of geometry, the same we now use in the high school, only slightly modified along algebraic lines. But this slight modification means an enormous gain in ease of presentation, as any one who has attempted to teach Euclid's theory of proportion to a class of young pupils can testify. In like manner algebra gains greatly in comprehensibility by the introduction of graphic, that is geometric, methods in many places, and text-book makers are beginning to realize this more and more. The two belong together in an educational scheme, and neither should be studied without continual reference to the other and illustration by the other.

When this scheme of carrying on the two side by side becomes general, I predict that the great preponderance of pupils who "don't like" (that is, don't understand) algebra, over those who "like" (understand) geometry will in a measure disappear. Not entirely, however, as the experiential, concrete conceptions of geometry will always appeal to a larger circle of students than the more abstract conceptions and processes of algebra. This preference for the concrete to the abstract is one of the fundamental qualities of the mind, and it seems to me to be a reversal of the natural order to make the pupil finish his algebra before he has the aid of geometry to help him over many hard places.

The theory that the development of the individual is an epitome of the development of the race lends strong support. Geometry was a full-fledged science while men were still doubtful about vulgar fractions, many centuries passed before the meaning of negative numbers dawned upon the human intellect, and a rational conception of an imaginary is scarcely a century old. Yet the negative comes to the surface as soon as the simplest equations are studied, and the imaginary appears simultaneously with the quadratic. Why then should it be a cause for surprise that the average fifteen-year-old schoolboy is so often hopelessly befogged over his algebra? And is it not sound pedagogy to give him a few geometric concepts before he gets very far into the subject so that he may have a little aid toward a better understanding of the subject matter and processes in his work?

SOME SUGGESTIONS ON THE TEACHING OF HIGH SCHOOL MATHEMATICS.

BY R. L. SHORT.

Assistant Professor of Mathematics, University of Illinois.

It is a conceded fact that in most technical schools mathematics furnishes a source of considerable annoyance to both the faculty and the student. The cause for this may be varied. It may arise from lack of application, lack of preparation, lack of analytic power either inherent or from neglect on the part of the pupil, the teacher or both. That we may have a remedy for this state of affairs, radical changes in high school methods must in many instances be made. In some parts of the state the idea prevails that mathematics is easily taught, that little or no special preparation is necessary. One city superintendent informed the writer that any one could teach algebra, that an expensive teacher for this subject was unnecessary. In a school where such ideas hold little can be expected of the pupil. Often the teacher may have ample academic preparation and be hampered by the rules of the school or by his own lack of knowledge of how to present the subject so as to increase the student's interest and to give him mathematical power.

It occurs to the writer that the following changes will aid in the teaching of the elements of this important subject: First, the speed is far too slow. In any branch of the subject there is a continuity, the appreciation of which is entirely lost unless rapid progress is made. While it is granted that knowledge is a function of time yet rapid progress in the subject matter and one or more reviews over the same ground will be far better than a slow rate of speed. In mathematics more than in any other subject is the development made step by step. The student sees no use for his rules for factoring until he reaches fractions. If a month or more intervenes between his study of factoring and of fractions he will have lost much of his grip on factoring. It is the application of the subject that makes mathematics interesting and helps fix rule and method in the student's mind. More attention should be paid to factoring and to the reduction of complicated forms and to giv-

ing the student an insight into the shorter methods of solution. Likewise in geometry one theorem a day gives no idea of continuity or of relation. One theorem fades from the boy's mind before he has occasion to use it. Double the speed at least and then the progress will be too slow.

Some will say that the student cannot prepare lessons of such length. That depends upon the high school. There are high schools where the student carries nine subjects, has nine recitations a day. That means he does nothing in any of them. Four or five subjects are all that should be allowed any one. A school that does more cannot do acceptable work. It has been the writer's experience that one can carry a beginning class *through* the work in such a text as Well's "Academic Algebra" or Wentworth's "High School Algebra" in thirty-six weeks and have plenty of time for review. With the period of fifty-four weeks allowed by most schools for the completion of this work a student ought to go over the entire work at least twice. The watchword of every teacher of mathematics should be *review, review, review*.

Second: The student must have more analytic power. He goes to college with his mind still in the experimental state. He does not think things out. He finds by trial whether this or that scheme will do. While it is not always possible to give a young student analytic methods, he can be taught to think and that is three-fourths of the battle. If the teacher will so plan his recitations and so formulate his questions that the student will be required to think, the power of analysis will soon begin to develop. Too many teachers seem to think because *they* know their subject that no time is necessary for preparation. The successful teacher plans his work the fortieth time, more carefully than he did the first time over. The solution of original propositions in geometry is a great aid in the development of the analytic faculty.

Third: More graphical methods should be used. They arouse interest and give a meaning to a jumble of algebraic symbols and expressions. It is acknowledged that the equation is the cause for the existence of algebra. Introduce the graph as soon as the equation is reached. Many abstruse parts will be made clear and simple by the use of this method. Reasons for the limited number of integral solutions in problems in indeterminate equa-

tions, why two linear equations have a single solution, why two simultaneous equations of second degree have four and only four solutions, etc., are all seen at a glance when the equation is plotted.

It is gratifying to note that the mathematics of the high school is improving, that the student of today knows more of what he ought to know than did the student of a few years ago. It is to be hoped that progress may spread rapidly and that this movement toward modern methods may soon reach all schools.

SOME RECENT DISCUSSION ON THE TEACHING OF MATHEMATICS.

BY W. W. BEMAN.

Professor of Mathematics, University of Michigan.

(*Continued from page 31.*)

7. In making men in any profession of applied science feel that they know the principles on which it is based and according to which it is being developed.

8. In giving to acute philosophical minds a logical counsel of perfection altogether charming and satisfying, and so preventing their attempting to develop any philosophical subject from the purely abstract point of view, because the absurdity of such an attempt has become obvious."

He believes most sincerely that these desirable functions would be performed well under the new system which is suggested. It may be well to quote a characteristic passage: "The ancients devoted a lifetime to the study of arithmetic; it required days to extract a square root or to multiply two numbers together. Is there any great harm in skipping all that, in letting a boy learn multiplication sums, and in starting his most abstract reasoning at a more advanced point? Where would be the harm in letting a boy assume the truth of many propositions of the first four books of Euclid, letting him accept their truth partly by faith, partly by trial? Giving him the whole fifth book of Euclid by simple algebra? Letting him assume the sixth book to be axiomatic? Letting him, in fact, begin his severer studies where he is now in the habit of leaving off? We do much less orthodox things. Every here and there in one's mathematical studies one makes

exceedingly large assumptions, because the methodical study would be ridiculous even in the eyes of the most pedantic of teachers. I can imagine a whole year devoted to the philosophical study of many things that a student now takes in his stride without trouble. The present method of training the mind of a mathematical teacher causes it to strain at gnats and to swallow camels. Such gnats are most of the propositions of the sixth book of Euclid; propositions generally about incommensurables; the use of arithmetic in geometry; the parallelogram of forces, etc.; decimals. The camels I do not care to mention because I am in favor of their being swallowed, and indeed I should like to see them greatly increased in number; these exist in the simplest arithmetic, and geometry and algebra. Why not put aside ever so much more, so as to let a young boy get quickly to the solution of partial differential equations and other useful parts of mathematics that only a few men now ever reach? I have no right to dictate in these matters to the pure mathematicians. They may see more clearly than I do the necessity of a great mathematician going through the whole grind in the orthodox way; but, if so, I hardly see their position in regard to arithmetic and other things in the study of which they do allow skipping. I should have thought that the advantage of knowing how to use spherical harmonics or Bessel functions at the age of seventeen, so as to be able to start in mathematics at Cambridge just about the place where some of the best mathematical men now end their studies forever, of starting at this high level with youthful enthusiasm, and individuality, and inventiveness, would more than compensate for the evils of skipping."

In the very interesting discussion which followed the reading of the paper and the presentation of the syllabus decided objection was made to the strongly utilitarian tendency of a large part of Professor Perry's remarks. Professor Forsyth, in particular, said: "I must point out what would be a platitude if we were not in discussion, that scientific subjects do not progress necessarily on the lines of direct usefulness. Very many of the applications of the theories of pure mathematics have come many years, sometimes centuries, after the actual discoveries themselves. The weapons were at hand, but the men were not ready to use

them. Take the case of medicine, which surely is a practical subject. It owes immense debts to the study of sciences like physiology and bacteriology; yet these have been developed and continue to be developed, along their own lines, without being guided in the direction of immediate application at every turn. Yet independent as has been their development, it is notorious that, perhaps all the more because of their freedom in growth, they have provided new knowledge that is of the utmost importance in the conduct of living processes. Take one last example, the x-rays. If any one had been set down, as a practical problem, to take a photograph through solid things, I think the common answer would have been that he was being told to solve an insoluble problem. Yet its solution came from the physicists, indirectly as it were, in the course of researches made to obtain knowledge for its own sake. The knowledge so obtained has subsequently led to wonderful results in its application. Influenced by these examples and by others more directly mathematical upon which I shall not enter, I must decline to accept utility as the main or the sole discriminating test, either in the study or the teaching of mathematics."

Had I the time I should like to quote from the remarks of several of the other participants, but I shall have to content myself with a few selections.

Lord Kelvin wrote: "I am overdone with work which must not be postponed, and I am sorry therefore not to be able to write anything on the subject. I think your syllabus was good indeed. It is very like the teaching I had from my father."

Sir John Gorst, chairman of the joint session, told a unique experience he had in New Zealand in his younger days. He said: "I taught, or attempted to teach, mathematics to the Maori boys and men. As far as the teaching of arithmetic went I taught on a sort of embryo Sonnenschein principle, and I found them remarkably apt and quick pupils. They learned the practical arithmetic, which was useful to them in actual life, and they learned it with extraordinary rapidity—far faster than boys or men would generally learn it in this country. But when in my youthful enthusiasm I proceeded to try to teach some of them Euclid, or rather geometry after the Euclid fashion, I absolutely and entirely failed. There was not one of them that could grasp or under-

stand the simplest of the propositions of Euclid. . . Had I had the advantage of the discussion to which I have listened to-day, I should have abandoned teaching in the ordinary way until they had been familiarized with angles, lines, areas, and geometrical figures, of which the Maori youth was absolutely ignorant. I suppose by a method of that kind even the least developed intellect of the uncivilized native of New Zealand might have been brought to take in some of the very simple propositions of geometry."

The frank confessions and the suggestions for improvement—which we must assume to be equally honest—found in these discussions (on Professor Perry's syllabus entitled: A Course of Elementary Mathematics) throw an unmistakable flood of light upon the mathematical situation in the public schools of England. Held in servitude by the boards of examiners, who will not permit a hair's breadth variation from the order of Euclid's elements, and required to submit to external examinations which serve principally to reveal the malovelent ingenuity of framers of impractical problems, both teachers and pupils groan under a worse than Egyptian bondage. Small wonder that elementary mathematical teaching in England has fallen so far below that of France, Italy or America!

The probable outcome of this discussion is hard to forecast. That the committee will recommend some substantial changes for the better may reasonably be expected. That it will advise tha rejection of Euclid as a textbook is by no means certain, though it would seem as if nothing pedagogically worse than the teaching of Euclid, without any knowledge of concrete geometry, or any use of rule, compasses and protractor, could easily be devised.

The process of change, even when once begun, will need at least a generation. Boards of examiners must "put away the evils of their doing from before all eyes and learn to do well."

Teachers will have to be trained by some other method than merely putting new wine into old bottles, and possibly the author in his anguish will be compelled to say, "Oh, that mine adversary had written a book!"

AIMS AND METHODS IN THE TEACHING OF PLANE GEOMETRY.

BY MABEL SYKES,
South Chicago High School.

A study of the methods pursued today by teachers of geometry in secondary schools, may perhaps reveal an evolution of which some of our friends have been unconscious. Formerly the only object seemed to be the reproduction and retention of the details of an elaborate system built up from supposedly fundamental axioms. General principles were not dwelt on and the originals introduced consisted merely of a set of unrelated puzzles, many of them requiring a large amount of invention and few of them of value save as gymnastics. We are learning today, however, that the object of our work should be ideas and principles, not details and gymnastics.

What should be emphasized in teaching geometry must be decided by the relation of the subject to life. Some of us have concluded that what the pupil requires is an acquaintance with the statements of certain fundamental theories and the power to apply them. This is true whether he goes on with his mathematics or requires his geometry in work that he may undertake after he leaves school. Illustrations are numerous and well known; they abound in the work of the carpenter, architect, engineer or draughtsman, while in more advanced mathematics one is liable to find that the conditions of one's problem or points omitted from one's text require the solution of an easy original. But aside from all applications of geometry the power to appreciate and to make a formal demonstration is of great value. Whatever may be one's calling the ability to stick to the subject, to answer a question directly, to make or intelligently to participate in a logical argument, are necessary even in ordinary conversation. We do not claim for a moment that geometry is the only subject in which this training can be obtained. Indeed every good teacher must have this in mind in every question asked, but we do claim that the pupil gets his first and often his only clear view of the logic of argumentation through geometrical demonstrations. We say his

first view—for a careful study of the equipment of the pupil at the outset, reveals the fact that although he is accustomed to draw inferences, the idea of the formal demonstration is new to him and is the cause of most of his early difficulties. We say his last view—for very often he does not go beyond the secondary school. Search where we may, however, we fail to find the first necessity for carrying in mind the details of text proofs.

It seems that the difference between poor and good teaching does not depend upon details of any sort, even such details as graphs, colored chalk or correlation with physics or drawing, but is a difference of emphasis. This has been so well said by Dr. John M. Coulter in a recent address that we cannot refrain from quoting a portion of it. We believe that it applies just as truly to mathematics as it does to botany or any natural science.

"In the one case the facts are presented in the helter-skelter fashion, solid and substantial enough, but a regular mob, with no logical arrangement, no evolution of a controlling idea. Details are endless, no emphasis brings out certain things into prominence and subordinates others, and the whole subject is as featureless as a plain, where the dead level of monotony kills off every one but the drudge. It is the spirit of analysis, a dead body of facts without a vitalizing spirit. In the other case fewer facts are presented, but they are the important ones and marshalled in orderly array, battalion by battalion, they move as a great whole towards some definite object. The facts may fade away, even the battalions may grow dim, but the great movement remains definite and clear as a memory which is an inspiration. Instead of a level plain, there are mountain peaks and valleys, there is a perspective and there are vistas from every point of view. This is the spirit of synthesis, which vitalizes the great body of facts and makes them glow. To the teacher, in his work of training, an unrelated fact is worse than useless."

It may be of interest to give some details of methods employed from this point of view, although we do not claim that they represent anything more than a groping after the ideal. To begin with, then, we depart from tradition at the outset. It will be seen that if the order of the text is followed reductio ad absurdum and superposition proofs are introduced before the direct

proof is even dimly understood and of direct proofs long and short, hard and easy will be indiscrimnately mixed. To avoid these difficulties pupils are occasionally allowed to learn the statements of certain theorems and temporarily to assume the proofs. Abandoning the order of the text, selected exercises are given to train the pupils to make direct demonstrations. In selecting these exercises the following points concerning demonstrations are kept in mind: (1) the pupil must understand clearly the relation between hypothesis and conclusion; (2) he must appreciate that all of his inferences come directly from the hypothesis and that he works directly for a stated result; (3) he must be taught how to apply previous theorems. Such geometrical problems are chosen for the exercises as will be most likely to appeal to pupils and as are applications of most important principles, such as the relations between angles formed when two parallels are cut by a transversal and directly dependent proportions, and the applications of equal triangles. These exercises are all carefully graded so that the easier are given first. Reductio ad absurdum and superposition proofs are given later.

Pupils are sometimes allowed to prove certain propositions by propositions that follow, provided that they are in no way interdependent according to the scheme given in the text used. For example, in most texts the theorems—two lines perpendicular to the same straight line are parallel; two lines parallel to the same straight line are parallel, and a line perpendicular to one of two parallels is perpendicular to the other—may be easily proved as corollaries of the propositions about parallel lines and general transversals and their converses. They are rarely so proved, however; more often they precede these theorems and then the proofs are needlessly difficult.

Again, if propositions appeal to pupils as almost self-evident, their proofs may be permanently omitted. For example, they will appreciate that a point and a direction determine the position of a line just as truly as do two points and after their attention has been called to this fact, they fail to see the necessity of proving that only one perpendicular can be drawn to a given line from a given point. We might as well acknowledge that our most carefully thought out schemes might, if we knew more mathematics,

appear less rigid than we had hoped, and that a few assumptions more or less at the outset on the part of the class are of very little consequence.

In general there are two methods to be employed, reproduction of proofs given in a text and invention of proofs by pupils. We believe in both. If the pupil is required to work out as originals all of the proofs given to the class many are encountered which require more invention than can reasonably be expected and the pupil becomes dependent upon the teacher. These proofs often illustrate to great advantage ideas and points of importance and should therefore be discussed. If the pupil is allowed to prepare them from a text, his attention is called to the important points, he learns to read mathematics and to study independently, while the form of the demonstration serves as a model for his own. This should be only a part of the work, however, for if the proofs given are selected with reference to some controlling idea and this idea carefully developed and explained many proofs given in the book will be proved by the class with very little assistance. This can be done by a sort of laboratory method. It will be found in the end that most pupils will prepare lessons as easily by one method as by the other.

Some may object that we lose sight of the sequence of the whole which has always been considered of the first importance in geometry. A little consideration will show that this is not so. We feel sure that the pupil is not mature enough at the outset to appreciate it, but he will appreciate it in the end if interdependent exercises are carefully avoided and the interdependence carefully explained and discussed whenever they do arise, as arise they will, and if reviews are carefully conducted.

In conclusion, we can only say that we firmly believe that if the teacher has controlling ideas and general principles in mind in selecting details and methods of presenting details, anything is allowable which serves his purpose; but that details must be selected and methods chosen with reference to their ideas and principles while everything that does not directly serve some purpose must be cast aside as rubbish. Thus and thus only can his work become effective.

Round Table.

When teachers of mathematics begin putting their class work on a practical basis, short methods, methods of checking computations, methods ot ascertaining the effects of erroneous data upon results become matters of great importance. To qualify for such matters teachers will find little help in the standard mathematical texts. The practice of the actual users of mathematics is the best source of information on these points. The following letter from Mr. Mogensen, one of the engineering experts of the Illinois Steel Company, of South Chicago, contains a number of practical suggestions in the directions mentioned. Coming, as they do, warm from actual use, they bear a glow of reality that will not fail to inspire interest in the mathematical class room.

ED.

METHODS OF COMPUTING USED BY ENGINEERS.

MR. EDITOR:

You have asked me to contribute to your *Round Table* some notes from my own experience on "short cuts" in computations or methods used by engineers to facilitate calculations. I am not prepared to treat the subject exhaustively, but if a few simple examples, that come readily to mind, have any value you are welcome to them.

"Short cuts" in computations usually consist in applying rules that are theoretically erroneous, but that are known to give sufficiently accurate results within certain limits. It will be readily seen, of course, that he who has the most thorough mathematical education and training is best qualified to use methods of approximation.

Before an approximate rule is applied to the solution of a problem the degree of accuracy required must be decided and the limitations of the rule must be considered. If, for example, it is desired to find the area of a circular segment of which the chord (c) and height, or rise, (h) are known, the work involved may be considerably reduced by making use of certain tables found in several hand books (Kent's, Trautwine's and others); but if the segment is not large enough to differ appreciably from a similar part of a parabola, it is readily seen to equal

$$\frac{2}{3} c\, h \text{ (nearly)}.$$

If the angle subtended by the chord is 60° the result is a little less than than $1\frac{1}{2}$ per cent too small and this error will increase or decrease as the subtended arc is increased, or decreased, respectively.

Supposing, now, that the above rule were used in calculating the

cubical contents of a body of masonry, worth \$10 a cubic yard, and that the error affected only 10 per cent of the volume. It is plain that the money value of the error would be very small, being in fact only 14 cents in \$100.

In land surveying, on the drawing board, and in other places it is often desirable to know the difference in length between the hypothenuse (a) and the base (b) of a rignt-angled triangle when the included angle is small, and the height (h) and either one of the other sides is known. A simple rule for this purpose is readily found when we remember that $a^2 = b^2, + h^2$ and that $a = (b^2 + h^2)^{\frac{1}{2}} = b + \frac{h^2}{2b} - \frac{h^4}{8b^3} + \frac{h^6}{16b^5} \ldots\ldots$

(By Bin. Theorem)

If, now, h is small enough in comparison with b, all but the first two terms of the second member of the equation may be neglected, and we have

$$a - b = \frac{h^2}{2b} -;$$

or, as a and b are of nearly equal lengths,

$$a - b = \frac{h^2}{2a} \text{ (approx)}$$

The same result can also be obtained geometrically from properties of the circle. This rule is well suited for rapid mental application.

If we assume that the length of the hypothenuse is 100 feet and that the height is 5 feet, then by the above rule the difference in the lengths of the hypothenuse and the base is 1½ inches, and this quantity, thus determined, is less than $^1/_{[illegible]}$ of 1 % in error. But here, again, the error increases as the height h is increased.

Applying this rule to the computation of the length of a sewer, costing \$5 a linear foot, and built on a 2 % grade between manholes, whose distance apart is known, we see that—

$$a - b = \frac{2^2}{2 \times 100} = 0.02.$$

That is, the sewer is $^1/_{50}$ part of 1 % longer than the horizontal distance between the manholes, and the money value of the error in assuming the sewer horizontal instead of on a 2 % grade, is 10 cents in 100 feet.

Again, if in a rectangular steel tank, 10 feet high, each of two adjoining sides has a batter of 1 foot, then the length of a steel angle, to which the side plates would be riveted at the corner, would be

$$10 + \frac{(\sqrt{1^2 + 1^2})^2}{2 \times 10} = 10.1 \text{ feet, say;}$$

and this result is well within the required accuracy.

Engineers, especially the younger men, are often required to make computations involving a large amount of tedious arithmetical work,

which can be considerably abbreviated by the use of Crelle's multiplication tables, of Fletcher's slide rule, or of some kind of reckoning machine. The work should, of course, never be needlessly extended, and this is especially true when it must be done in the ordinary way—with paper and pencil. In any case, it is very necessary to have at hand some simple means for checking the results obtained.

Consider the product,

$$1.234 \times 56.789$$

If we use the ordinary long multiplication, we get a quantity containing six decimals. If, now, sufficient accuracy is secured when three decimals only are retained, we might profitably employ shortened (contracted) multiplication, thus:*

$$\begin{array}{r} 56.789 \\ 432.1 \\ \hline 56789 \\ 11358 \\ 1703 \\ 227 \\ \hline 70.077 \end{array}$$

thereby avoiding unnecessary work.

A very important subject is the matter of checking, and an exceedingly valuable aid for verifying the results of multiplication and division is the Mannheim slide rule—a simple tool with which it is well worth while to become thoroughly familiar. A little practice will enable anyone to make computations with this slide rule that are not more than from ¼ to $^1/_{10}$ of 1 % in error.

Ordinary multiplication and division can also be conveniently checked by the method of "casting out the nines," which, I suppose, is taught in all primary schools.

It is needless to say that computed results will always be doubtful unless the mind is persistently trained to accuracy in dealing with numerical quantities.

It should be added that engineering computations are facilitated very greatly by the use of a multitude of mathematical tables, diagrams and rules that are to be found in hand books, of which the engineer usually has several at his disposal.

But my notes are becoming somewhat lengthy, and perhaps you have had enough of them. Sincerely yours,

PETER MOGENSEN.

* Digits of multiplier written in reverse order, units' digit in the multiplier being placed under that decimal place of the multiplicand whose order is the same as the order of the last place to be retained in the product.

THE SLIDE RULE.

When our class in higher algebra was studying the subject of logarithms, they constructed out of coördinate paper two identical, uniformly graduated scales running from —100 to +100. The scales were so made that when the edges bearing the graduations were put side by side, the numbers increased from left to right. The two scales in this position formed a slide rule by means of which two numbers, either positive or negative, could be either added or subtracted.

The construction and use of this scale could very well be introduced with the negative number in algebra, or with the directed line in trigonometry.

A logarithmic scale was then constructed. To do this a scale was first graduated uniformly from 0 to 2. The logarithmic scale was then graduated from 1 to 100 in such a way that when the graduation 1 was placed at the 0 of the first scale, the reading of the first scale at any point of the second was the logarithm of the number indicated at that point on the second scale. Another logarithmic scale exactly like the first was then made. These two scales placed side by side formed the essential part of a slide rule by means of which multiplication and division could be performed.

When the pupils had thus made and learned to use this rudimentary slide rule, each one was provided with a cardboard slide rule which he mounted for his own use. The latter were obtained of Prof. P. B. Woodworth of Lewis Institute, for two cents each.

Chicago Manual Training School. WILLIAM O. BEAL.

MATHEMATICAL TEACHING SHOULD BE USEFUL.

Perhaps the main reason why modern mathematical teaching has wrung and is yet wringing the most caustic criticism from the sanest authorities in all the leading vocations and professions of modern society originates in the almost fanatical insistence by early mathematics teachers upon a degree of rigor which is altogether in advance of the possibilities of their pupils, and which is altogether antagonistic to the tendencies of modern social, industrial and scientific evolution. In brief, we are pretty generally regarded by the mathematical laiety as about the best preserved specimens of mediaeval scolasticism now extant. The classics have been vitalized very considerably by an infusion of modern method and spirit; but mathematics still stands aloof from the current of modern pedagogy with an air of supercilious disregard for the needs of man and with a dignity so rigid as to amount to about ten points conceit to one point of merit.

It is unfortunate that our accusers have too much of a show of reason in their charges and the only way in which their strictures can be deprived of their pertinency is by a summary rupture with archaic ideals and by a sincere and persistent effort on the part of teachers of mathematics to put their students into intelligent possession of every modern agency which will bring them more positively, more sympatheti-

cally and more efficiently to enter into the life of their time. The cloistral spirit, epitomized by the lofty unconcern of the teacher as to whether his subject is ever to be of value to anybody, must be given a decent burial among the dead dogmas of a respected past. Let the interment be so safely performed that the ghost may never again break forth amongst us to terrify us with the thought that we are degrading ourselves by making our instruction useful rather than cultural!

TRY THIS ONE ALSO.

When your boys and girls fail to give due regard to the distinctions between equations and inequalities and particularly when they manifest that persistent, but pernicious, tendency to apply the laws of the equation indiscriminately to the expression of inequality, give them the following puzzle and require them to find "the nigger in the wood pile":

Write the identity:

(1) $\frac{1}{2} = \frac{1}{2}$.

Take the logarithm of both sides, obtaining:

(2) $\log \frac{1}{2} = \log \frac{1}{2}$.

Evidently we may write:

(3) $3 > 2$.

Multiplying (2) and (3) member by member, we have—

(4) $3 \log \frac{1}{2} > 2 \log \frac{1}{2}$.

Passing to exponents:

(5) $\log (\frac{1}{2})^3 > \log (\frac{1}{2})^2$.

But as when the logarithm of the first of two numbers is greater than the logarithm of the second, the first number is greater than the second, we have from (5):

(6) $(\frac{1}{2})^3 > (\frac{1}{2})^2$;

or, what is the same thing:

(7) $\frac{1}{8} > \frac{1}{4}$.

(7) is perhaps absurd enough, but multiply both sides of the inequality by 8 and have:

(8) $1 > 2$.

(8) is certainly sufficiently surprising even for children. Where is the flaw in the argument?

Teachers should see to it that an error in a chain of reasoning which results in an absurdity be clearly brought out or students may come to think that all mathematical reasoning is mere "hocus pocus" to grind out any sort of conclusion desired by the puzzlist. Clarifying and systematizing muddled conditions is a valuable exercise for the student; but muddling things merely for the sake of the muddle is positively bad.

M.

CORRECTION.

For reasons not fully known to the editors, Professor Perry, of England, will not give the course of lectures announced in our April number at the University of Chicago. Professor Perry has decided not to come to America this summer.

Vol. III.] MARCH, 1904. [No. 9.

School Science

A Journal for Science Teachers.

EDITED BY C. E. LINEBARGER.

CONTENTS.

(*Continued on page 2 of cover.*)

SCHOOL SCIENCE, 2583 HERMITAGE AVENUE, CHICAGO.

Published Monthly, October to June, inclusive.

Price $2.00 Per Year. 25c. Per Copy.

Entered October 1, 1902, at Chicago, Ill., as second-class matter, under Act of Congress of March 3, 1879.

CONTENTS CONTINUED.

School Science

A Monthly Journal of Science Teaching

TERMS AND CONDITIONS OF PUBLICATION:

PRICE. The subscription price is Two Dollars a year, payable in advance; single copy, 25 cts

SCHOOL SCIENCE is sent free of charge to members of The Central Association of Science and Mathematics Teachers. Information as to the conditions of membership will be sent on request.

DATE. SCHOOL SCIENCE is issued about the first of the month, except during July, August and September.

POSTAGE IS PREPAID for all subscriptions in the United States, Canada and Mexico. For all other countries in the Postal Union, add 30 cents for postage.

CHANGE OF ADDRESS. Subscribers should promptly notify the publishers of any change of address; otherwise they are held responsible for magazines sent to their former address, until ordered stopped or address changed.

DISCONTINUANCES. When so ordered, the magazine is stopped at the expiration of the subscription. Without distinct orders to the contrary it is continued, as it has been found by experience that such is the wish of most of our subscribers.

Address all correspondence and make remittances payable to—

The School Science Press

Ravenswood, CHICAGO, ILL.

APPARATUS OF INTEREST TO TEACHERS.

The instrument shown in the illustration is especially well adapted for use in college and high school work. It is arranged with three tubes; a telescope with a hand adjustment with an aperture of 20 mm. and a focal length of 122 mm., giving a magnification of 6 diameters. A collimator tube that has the same dimensions is provided with a comparison prism and adjustable slit. The separate tube contains the scale, that may be reflected into position over the spectrum. The prism used is one

of 60°, made of a dense silicate flint glass, and has a 30 mm. aperture. This is contained in a light tight brass case, and has a specific gravity of 3.87. The angle of dispersion C — F is 1° 56′. The refractive index is 1.65, and the apparent separation of the D lines 6′.

We are prepared to quote a special price on this instrument, and can furnish other styles suitable for the highest class of work. We would be pleased to send a copy of our latest catalogue, illustrating this and other new pieces of apparatus, to any teachers who may be interested. C. H. Stoelting Co., successors to The Chicago Laboratory Supply and Scale Co., 39 W. Randolph street, Chicago.

1

S

RS

rnia.

S. C.

linn.

lass.

City.

City.

OW

S

.00

.00

.00